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1. Any theory strikes a balance between simplicity, accuracy, and generality. A theory may outlive its usefulness, and its adherents mistake its simplifications for the reality. They take the limits of their vision for the limits of the world.

2. General equilibrium theory, the core of economics, has reached this stage of obsolescence. Historically useful in enabling economists to get a grasp on the economic progress as an integrated system, its defects are now retarding further progress.

3. General equilibrium theory is to be replaced by a new paradigm, an interrelated set of ideas encapsulated under the headings friction, capital theory, natural selection and uncertainty.

4. Fric. Is an equilibrium theory, a defect in that it does not take time seriously. Only a small fraction of economic goods are alive at any time. There is no common market in which all participate and there cannot be one. There is only a series of shifting markets over time, loosely linked by uncertain expectations and by the consignment of previous investment decisions. and reproductive, distinguished commodities by location

5. There is a version of general equilibrium theory that formally distinguishes commodities by date (bread today is not bread tomorrow), so that one gets an intertemporal "equilibrium". This is a good example of sweeping the real problems under the rug: The assumptions required — perfect information, ~~market~~ strong common markets — are not realized in the world, and cannot be realized. A similar comment applies to

6. G-E Theory is not really very general. It neglects frictions. Since frictions appear most clearly in a spatial context, one may say the theory does not take space seriously.

7. Frictions refer to the cost of transactions, transportation and communication, and also to delays in the attainment of equilibrium (cf. friction of space, barriers to communication, frictional unemployment). There is no sharp dividing line between frictions and costs in general, which comprises the three categories of resources (used up, depleted or dilapidated), time (delays or resources tied up), and pain.

8. Frictions are in large measure responsible for the existence of the following: money (reducing transactions costs), the credit banking & credit system (ditto for transactions entailing future repayment) - the entire financial sector for that matter; cities (reduce friction by bringing masses of people & resources into proximity), large organizations (established communication channels), white collar and office work, the information-processing industry, the trade and transport sector, sales promotion & advertising, the postal service, the telephone system, and large parts of the legal system and government administration. General equilibrium is notoriously inept at incorporating this phenomenon.

9. Frictions have a radical philosophical consequence: the "individuation" of all things. No two points or locations are interchangeable, since one must take effort to go from one to the other. Each point of space is then a unique "commodity". But more: any two things, even if identical in form, occupy different locations, hence inherit their individuality. (Two loaves of bread on the same shelf are not interchangeable,

hence the injunction to grocers to rotate their stocks). It follows further that any statement about the world, even any act of perception, involves some degree of aggregation over diverse entities. The way to keep all this straight is by using measures, as expounded in Economics of Space and Time.

10. Frictions give everyone some degree of monopoly power by insulating them to some extent from substitution by a competitor. Hence perfect competition must fail as an accurate description of the world, even in principle. The "law of one price" is false, due to frictions. Never Another mainstay of general equilibrium falls. (On the other hand, existing theories of imperfect competition for the most part also ignore frictions.)

11. Some theories ignore frictions by making them absolute. Standard international trade theory postulates infinite ~~costs on~~ but zero transport costs for factors (as well as zero costs for products). Both extremes are wrong and again constitute a tradeoff of accuracy for simplicity. An echo is found in general equilibrium with its atomistic individuals cut off from all relations with each other except for exchanges.

12. In competitive equilibrium no activity makes a profit (costs ≥ revenues, with equality if the activity is running). But not so with frictions. We are discovering profitable opportunities all the time because we have unique advantages in some directions, and our insulation keeps this advantage open for some time.

13. There are also "internal" frictions, within a person, imperfect memory being one symptom. The problem of communicating with my future self is ~~the~~ analogous to communicating with another person. Information processing — thinking — is costly in terms of effort, time and worry. Given all this, it is at least dubious whether a person's behavior can be described as the maximization of a utility function. The difficulty is the same as that of describing an organization by a single utility function. Changes of mood, the prevalence of habits and rules of thumb are what one would expect instead.

14. Now for capital theory, the next peg of our new paradigm. By its nature, it takes time seriously. It also allows for an arbitrary degree of heterogeneity in capital goods and can incorporate frictions, so it also takes space seriously. Further, the concept of capital has widened progressively. From capital goods per se, the ideas of capital theory have been long applied to land, and natural resources, and then latterly to human beings themselves: the "human capital" approach. Thus capital theory bids fair to being a universal theoretical scheme.

And with this, consumer goods also become capital.

15. Much of the human capital literature is devoted to calculating rates of return for investments in education, medicine, migration, etc. to evaluate policies. There is a general principle that transposing questions from the normative to the positive sphere brings powerful results. The interesting question for us then is whether this approach can explain births and deaths, the building up and wearing down of the human machine, by thinking of people as (usually self-owned) capital goods literally.

16. With capital, value theory now re-emerges as the central core of economics. Can we then speak of a capital theory of value replacing the defunct labor theory? Yes, but with two provisos to avoid misunderstanding. First, it does not mean that capital goods are more important than people: all resources, natural, fabricated, and human, are at the semi encrypted level. And quantitatively, by all measures the human capital stock outweighs all other categories combined. Second, the theory does not consist of a basic yardstick to measure the "quantity of capital" in things, as people tried to do with labor. Rather, value theory derives a network of relations between inflow and outflow values, rents and present values, etc., which together form a coherent system.

17. Start with the relation: The value of something equals the sum of the stream of values it will create in future production, properly discounted. Is this correct? Only if we include "non-pecuniary" production. This includes contributions to enjoyment and relief of suffering, but can include motives of display, pride, vengeance and spite. ("Non-pecuniary" means paid values not contributing to further production). In this form the principle is (almost) tautologous. But note two things. First, values inevitably summarize future expectations. If these are uncertain, probability must enter in an essential way. Second, the value of goods produced in the future derives in turn from what they will produce in the still further future — so that we have a chain of expectations stretching in principle to infinity.

18. We now write down these relations. There is a remarkable convergence here of capital theory, control theory, Hamiltonian dynamics and national income analysis. In particular, in any dynamical system, with or without controls, and with or without an objective function, certain "dual" values appear which mimic the categories of wealth, income, prices, rents, land values and interest rates. The only slight exception modification is in the income concept. Write $Y = C + I$, "income = consumption + investment. It is well recognized that C is a proxy for welfare, enjoyment, etc. (and not a very accurate one. Most of the "paradoxes" in income measurement arise from this fact). We will replace C by U, which summarizes all "non-pecuniary" production — i.e. it "leaks" from the system and is not used in future production.

19. Let $x = (x_1, \dots x_n)$ be the state variables. This can refer to an entire economy, or an individual's domain, or something in between. (Actually, x should be a measure, but then dx/dt requires knowledge of Schwartz distributions). Let $v = (v_1, \dots v_m)$ be the control variables, if any. $U(t, x, v)$ is the rate of non-pecuniary production at time t in state x with control v. (All influences or discounting factors not included in x or v are captured in t). Dynamics are given by

$$\frac{dx}{dt} = g(t, x, v). \tag{1}$$

(This is not the most useful formulation even in this deterministic case, but it brings out the accounting concepts clearly). Consider the problem: Maximize $\int_{t_0}^{\infty} U(t, x, v)\, dt$ (2)

subject to (1), starting at t_0 in state x_0.

Let $w(t_0, x_0)$ be the value of this program. We interpret w as wealth.

Let $p_i(t,x) = \frac{\partial w(t,x)}{\partial x_i}$ $\quad (p_1, \dots p_n) = p$. We interpret p_i as the price of component x_i (Price is the increase of wealth occasioned per unit rise of x_i). $p \frac{dx}{dt}$ is then naturally investment, so that $y = U + p\frac{dx}{dt}$ is income.

The value of the marginal product of x_i is

$$VMP_i = \frac{\partial U}{\partial x_i} + p \frac{\partial g}{\partial x_i}, \tag{3}$$

the sum of the non-pecuniary and pecuniary products, respectively.

Theorem:

$$VMP_i + \frac{dp_i}{dt} = 0 \tag{4}$$

Proof: $\frac{dp_i}{dt} = \frac{d}{dt}\left(\frac{\partial w}{\partial x_i}\right) = \frac{\partial^2 w}{\partial x_i \partial t} + \sum_j \frac{\partial^2 w}{\partial x_i \partial x_j} \frac{dx_j}{dt}$

$$= \frac{\partial}{\partial x_i}\left[\frac{\partial w}{\partial t} + \sum_j \frac{\partial w}{\partial x_j}\frac{dx_j}{dt}\right] - \sum_j \frac{\partial w}{\partial x_j}\frac{\partial g_j(t,x,V)}{\partial x_i}$$

(where $\frac{dx_j}{dt} = g_j$ the j-th component of g)

$$= \frac{\partial}{\partial x_i}\left(\frac{dw}{dt}\right) - \sum_j p_j \frac{\partial g_j}{\partial x_i} = -\frac{\partial U}{\partial x_i} - p\frac{\partial g}{\partial x_i} = -VMP_i$$

(since $\frac{dw}{dt} = -U$ by definition of w). QED

Equation (4) is rather important for two reasons. First, writing it out as

$$\mathcal{H}(t, q, v, p) = U(t, q, v) + p\,g(t, q, v)$$

we note that (4) reads

$$-\frac{dp}{dt} = VMP = \frac{\partial \mathcal{H}}{\partial q}$$

$$\frac{dq}{dt} = g = \frac{\partial \mathcal{H}}{\partial p} \qquad (5)$$

form (3). Also,

But these are the Hamiltonian canonical equations for the control problem (1), (2), where $\mathcal{H}$ is the Hamiltonian. (The remaining conditions are that v is chosen at each time to maximize $\mathcal{H}$, which also makes economic sense, and terminal conditions which we needn't discuss).

Second, (4) makes direct economic sense. In integrated form it reads

$$p_i(t) = \int_t^\infty VMP_i(\tau)\, d\tau$$

which says that p_i is the present value of the stream of future marginal products of q_i (what about discounting? The discount factor is already built in to VMP_i — in other words we are dealing with discounted values throughout).

(In Hamiltonian dynamics, the q would be generalized coordinates, the p generalized momenta, $\mathcal{H}$ the Hamiltonian — generalized energy — and W Hamilton's function. It is remarkable that even the standard p, q, W notation carries over).

20. We continue the discussion, introducing rent, land value and interest rates. To make economic sense, the q must be interpreted as stocks.

Wealth is W, while PQ is the value of all stocks. This need not be equal. There is one more "entity" which can have value, and that is land — or rather space, since stocks of natural resources, soil, etc. can be put among the q_i. Competition for the privilege of occupying a portion of the physical universe yields rent, which capitalizes into land value L. Thus

$$W = PQ + L.$$

Next, Y as defined above is total product. This should somehow equal the earnings of the factors of production. Now q_i earns VMP_i per unit, so total earnings of all stocks is $VMP \cdot q$. This should differ from Y by land rent R. Thus

$$Y = VMP \cdot q + R$$

For consistency, L should be the present value of R. And so it is.

Theorem: $$L(t) = \int_t^\infty R(\tau)\, d\tau$$

proof: We prove the differentiated version, $R + \frac{dL}{dt} = 0$.

$$R + \frac{dL}{dt} = Y - VMP \cdot q + \frac{dW}{dt} - P\frac{dq}{dt} - q\frac{dP}{dt} \cdot q = 0,$$

since $\frac{dW}{dt} = -U = P\frac{dq}{dt} - Y$, and $VMP - \frac{dP}{dt} = 0$ QED

Finally, interest rates. But which? Nominal interest rates with the above price system are in effect 0, since all prices are in discounted terms. But real interest rates make sense if we can figure out how to measure the rate of inflation in the discounted prices. For,

$$\text{real rate} + \text{inflation rate} = \text{nominal rate} = 0,$$

so the real rate is just minus the inflation rate.

Now it is widely held that the weights used in calculating price indices are largely arbitrary.

We calculate the inflation rate using stocks as weights. (The alternative, universally used in practice, of using flows has the disadvantage that some weights must be negative, since some stocks are falling while others rise). The answer does not depend on the unit used to measure the fixed "stock" of space, so we set this to 1. Thus

$$\text{rate of inflation} = \frac{\frac{dp}{dt}q + \frac{dL}{dt}}{pq + L} = \frac{(-VMP)\cdot q - R}{pq + L}$$

from which we get the remarkable conclusion

Corollary: the real interest rate = income / wealth

Proof: the denominator = W, and the numerator = $-Y$ QED

21. The role of space in solving the "adding-up" problem, equating total income with total earnings, should be noted. Rent R is the balancing item, but how do we know that this equals the VMP of space? First, does space have a marginal product

in any sense? Suppose the answer is yes for a small portion of space in the neighborhood of any point s: VMP(s) dA = r(s) dA for an element of area (or volume) dA around s. Then VMP for all space is $\int r(s)\,dA$. Now various alternative uses compete for dA, which raises their net productivity by letting them rearrange their resources in a slightly more efficient less congested pattern. The high bidder determines r(s)dA, which is its contribution to net product. Thus r(s) makes sense. But is it true that

$$\int r(s)\,dA = R,$$

the balancing rent? A quick review of the possible justifications runs as follows:

a) constant returns to scale. This must be rejected at once. First the concept is not even well defined, due to the individuality of locations and their contents: shapes and arrangements matter; second, there are neighborhood effects, threshold effects, critical masses, etc., which would falsify it even if the concept could be clarified

b) locally constant returns, so that things balance at equilibrium (e.g. low point on the AC curve in perfect competition). This is basically correct, but the argument must involve space. Tentatively, try this:

i) If $Y < VMP \cdot q + \int r\,dA$ there are local increasing returns. This is brought about by net positive neighborhood effects, so resources tend to crowd together more (use less land). This reduces r (there may be higher rents at the center of crowding in, though less overall) and VMP may rise at the beginning of this process)

ii) If $Y > VMP \cdot q + \int r\,dA$ there are local decreasing returns. There are net negative neighborhood effects, so resources want to

spread out more (use more land). This raises r (though it may lower r at some points) and also raises VMP as resources interfere less with each other.

2. The relation of space and technology deserves further elaboration. Think of the components q_i as ranging over different resource types at different locations. There is a neighborhood effect from site A to site B if $\frac{\partial g_i}{\partial q_j} \neq 0$, where $\frac{dq_i}{dt} = g_i(t, q, v)$, q_i is located at B and q_j at A. (Neighborhood effects are also called positive physical externalities. It is positive or negative (or $\frac{\partial g_i}{\partial q_j} > 0, < 0$, respectively). In value terms, look at the sign of $p_i \frac{\partial g_i}{\partial q_j}$, since p_i may be < 0)

Roughly, the intensity of these effects varies inversely with distance, but the correlate is low, and partially controllable by barriers.

Define scale change in the intensive sense as a multiplication of all q_i (in a region) by a constant. ("Space" is not multiplied by this constant). The absence of any neighborhood effects would yield the principle of superposition (so that scale returns are not only constant, but linear) In this case, $r = 1$.

(fix the diagram, include movement)

Congestion refers to short-range externalities, inversely negative in the value sense for sufficiently high densities. Pollution refers to long-range externalities, which may be positive or negative. (The term pollution is used in other senses too - as contamination by foreign substances.)

(This is the "law of diminishing returns".)

Any particular configuration might have net positive or negative externalities, creating negative or positive rents, respectively. There is, however, an additional universal attraction among things to reduce the

costs of transportation ~~change~~ flows between them. Thus densities will rise until counterbalanced by negative externalities. This explains why rent is (almost) universally positive, and why we think of high density as usually creating "bads" rather than "goods".

23. The foregoing exposition of capital theory, control theory and national income analysis raises more questions than it answers. Whose objective function U are we talking about? To what does it correspond in the real world? Who is controlling the system? Do prices correspond to (discounted) real-world prices, or are they merely "shadows"? Whose wealth is represented by w? Which interest rate is "the" real one? What role is time, per se, playing in this system? Whose income is Y? What about frictions? What about uncertainty about non-deterministic systems and uncertainty in general?

24. The rest of this paper will be devoted in effect to answering these questions. As a start, these prices cannot be identified with real-world prices. The latter do not always exist, and are subject to monopolistic and other distortions due to ~~frictions~~ imperfect markets (All of these effects arise from frictions). Not all (value)-externalities are accounted for, and ~~prices~~ imperfect forecasting produces still other anomalies (again, these effects are due to frictions - costs of negotiating, and of processing information)

Further, society does not seem to be optimizing any objective function in its development. (Nor even do individuals do so, we have argued above).

25. There is, nonetheless, a close connection between the real world and the control model, at least two levels. First, note that as personal forces

which lead real prices to approximate the behavior of the shadow prices, and lead to the emergence of behavior approximately the existence of some "social objective function". (The degree of approximation is worse, the greater the friction level). Second, actual prices are themselves a factor of the economy (one themselves among the components of the state variable, & (and hence themselves carry shadow prices P)

26. Capital theory has a striking connection with the welfare aspects of general equilibrium theory, specifically the conditions for a "welfare optimum". The heart of the connection is the present value formula

$$P_i(t) = \int_t^\infty VMP_i \, dt = \int_t^\infty \left[\frac{\partial U}{\partial q_i} + p \frac{\partial g}{\partial q_i} \right]$$

First, if VMP_i there are no value externalities: All the effects of q_i on all the state variables are taken into account in $\frac{\partial g}{\partial q_i}$, as well as the non-pecuniary effects $\frac{\partial U}{\partial q_i}$. Second, there are no monopolistic distortions; the effect of q_i on P, though they may exist, are not counted. Finally, P_i reflects fully the contribution of q_i over all future time.

These conditions replace the general equilibrium conclusion that a Pareto optimum can be realized as a competitive equilibrium. As it stands the proposition is essentially tautological, first because perfect competition does not (can not) exist (see above) and second because market optimality is a trivial notion, as will be discussed later. Since a tautology cannot imply an impossibility, the other premises of the theorem must be false — convexity assumptions or technology and preferences must be false in the real world.

27. The role of interest rates and resource returns should be noted. All resources - human and non-human - are treated symmetrically. The wage rate is the VMP of labor. (A distinct one, in general, for each type, location and date of labor). The VMP of a capital good is the rent of that good, not the interest rate. Interest is no more closely connected to capital goods than it is to labor. It is therefore false that lower interest rates lead to greater capital intensity (contra Hicks value of Capital; Ferguson, neoclassical Theory. The Stretto-Samuelson trade-off between wage rates and interest rates is fallacious.

28. To digress a bit, the role of interest in the real world, suppose the (real) interest rate falls, then one will want to employ more resources - both human + non-human - to the point where the marginal rate of return falls to the lower rate. Total resources being given at any moment, one may say that the real interest rate tends toward the level that generates full employment of all resources. (Relative prices of the different resources adjust at the same time so that the markets for each individual will clear).

If frictions are now added in, we have the basics for a theory of macroeconomic fluctuations. First, with frictions, full employment should be replaced by the "natural rate of unemployment". Second, with frictions in the "money market (or "capital" market) — say lags in adjustment — resources will be either (excess) unemployed (r too high) or "over employed" (overtight markets) with r too low. It is false that depressions are primarily a phenomenon of unemployed labor: capital goods are unemployed to at least the same extent ("excess capacity") (This begins critique of the "classical" prescription to reduce wages is individual, as well as his emphasis on expectations, marginal utility, "capital" and interest.) But his main theory of interest is false,

and his emphasis on the conscious (unconscious misplaced)