Faden | Econ 601 Advanced Theory | October 1978

NOTES ON MICRO SOCIAL ACCOUNTING AND MULTIPERIOD BUDGETING

These notes expound some concepts which are fundamental for economic theory in general and the rest of this course in particular.

An economic agent can be a person, a household, a corporation, a non-profit organization, a government agency, etc. - any entity which may be thought of as acting as a unit, choosing among available opportunities, and possibly having a preference ordering.

This raises a question: if a single agent plays several roles, to what extent is it justified to look at each role in isolation? E.g., the same person may be head of a household, manage a firm, and contribute to political parties. Is it justified to think of this one person as a utility maximizer in one role, a profit maximizer in another, and a chooser of public goods in a third?

We need a concise way of describing the economic status of an economic agent, to get a handle on the opportunity set available to him. Consider the balance sheet of an agent as of a given moment of time. This is a list of his assets and liabilities, measured in physical quantities and in money values. The basic accounting identity is (in value terms):

Assets - Liabilities = Net Worth,

or A - L = NW. For our purposes there is a better classification. Distinguish physical assets (PA) from financial assets (FA). The former include such items as land, structures, equipment, vehicles, furniture and inventories. The latter include money, savings accounts, bonds, receivables, charge accounts, mortgages, accrued wages, vested pensions, etc. Basically, a financial asset is a claim on some other economic agent, while a physical asset is a tangible entity on which one has ownership rights (= your right of access to the entity, and the right to exclude "trespass" by any other agent).

Here are some borderline cases: Gold and jewelry are physical assets. Corporate stock is in practice a (variable-yield) financial asset for large publicly-held firms, but an indirect way of holding the physical assets of the firm in the case of closely-held firms. (Holding companies and corporate pyramids, in which some corporations own the stock of others, may be disentangled by input-output analysis.) The "human capital" approach notes that a person's own body (and mind) is one of his physical assets.

The things owned by an agent (i.e. his physical assets) may not coincide with the things <u>controlled</u> by that agent. This occurs in two broad ways: by <u>rentals</u> -- A relinquishes possession of a good to B's use for a period in return for a rental payment (in the employment relation the good is A's own body, and the rental payment is the wage); and by <u>bailments</u> -- where B performs a service on the good for which A pays B (e.g., repair services, surgery, transportation; the stockholders of a firm may be thought of as collectively bailing the corporate assets into the custody of the management team).

There are all sorts of partial ownership relations which can in principle be resolved into rights and claims among different agents: patents, franchises, easements, office holding, trusteeship, citizenship. Goodwill and reputation appear not to be separate assets, but values embodied in other assets (just as the value of any asset is a reflection in part of the demands of other agents).

We will ignore the complications suggested by these last paragraphs, and merely note that an adequate theory still awaits formulation.

Liabilities are precisely the obverses of financial assets. That is, a financial asset on one agent's balance sheet is exactly matched by a liability on some other balance sheet. (Example: bondholder and bond-issuer, depositor and bank, creditor and debtor, currencyholder and central bank.) Suppose there are N economic agents in the economy, and let B_{ij} be the gross amount that agent j owes to agent i. Let FA_i and L_i be i's financial assets and liabilities. Then

$$FA_i = \sum_{j=1}^{N} B_{ij}, \quad L_j = \sum_{i=1}^{N} B_{ij},$$

and, for the entire economy,

$$FA = \Sigma_i FA_i = \Sigma_i \Sigma_j B_{ij} = \Sigma_j \Sigma_i B_{ij} = \Sigma_j L_j = L.$$

<u>Definition</u>: the <u>net creditor position</u> of agent i, $B_i = FA_i - L_i$

It follows that $\sum_{i=1}^{N} B_i = 0$. The balance sheet identity may be rewritten

$$NW_i = PA_i + FA_i - L_i = PA_i + B_i.$$

Adding over all agents yields NW = PA,

or: total wealth = total (physical) ownership.

(Note: <u>all</u> agents must be included - governments, central banks, and foreigners).

<u>Consolidation</u>: Let S and T be two disjoint sets of agents. The total gross debt of T to S is

$$\sum_{i\varepsilon S} \sum_{j\varepsilon T} B_{ij}$$

The consolidated balance sheet for S is obtained by pooling all the assets and liabilities of $i \varepsilon S$ (cancelling any B_{ij} for which $i, j \varepsilon S$). This yields $NW_S = \sum_{i \varepsilon S} NW_i$,

$$PA_S = \sum_{i \varepsilon S} PA_i \ , \ B_S = \sum_{i \varepsilon S} B_i$$

In principle this is the way to obtain the economic status and relations among countries, regions, social classes, etc.

So far we have dealt only with money values. This can usually be broken down further into a sum of price-quantity products, and this is essential for time comparisons.

Distinguish three kinds of value yardsticks:

(1) current, or nominal dollars (the actual market values);

(2) constant, real, or deflated dollars (let $\pi(t)$ be a price index, $P(t)$, $V(t)$ a current price, value; then $P(t)/\pi(t)$, $V(t)/\pi(t)$ are deflated prices, values);

(3) discounted, capitalized, or present-value dollars (this is defined only in terms of a stream of discount or interest rates, let $Z(t)$ be the value at t of $1 at time 0 invested at these rates; then $P^*(t) = P(t)/Z(t)$, $V^*(t) = V(t)/Z(t)$ are discounted prices, values);

The rate of inflation is $\frac{d\pi}{dt}/\pi$, while $\frac{dZ}{dt}/Z = i$ is the instantaneous rate of (nominal) interest, in continuous time. $i - \frac{d\pi}{dt}/\pi$ is the real rate of interest.

In making comparisons over time, we may subtract current, constant or discounted values, getting three different answers. Thus, the following equations have three different versions.

Definition: <u>savings</u> = change in net worth $S = \Delta NW$

Definition: <u>investment</u> = change in physical assets $I = \Delta PA$

Definition: <u>net lending</u> = change in net creditor position ΔB

For any agent i, we have $S_i = I_i + \Delta B_i$ Adding over all agents yields S = I, the basic macroeconomic identity. Writing $V = \Sigma_j P_j x_j$, a price-quantity breakdown, we get $\frac{dV}{dt} = \Sigma_j \frac{dP_j}{dt} x_j + \Sigma_j P_j \frac{dx_j}{dt}$, separating value change into that due to price change (capital gains) and quantity change.

Note that for a single agent or sector, capital gains $\neq 0$ in general even in real terms, since one may hold assets whose prices rise faster than prices in general. A sensible way to <u>define</u> the price index π is to make capital gains = 0 for the economy as a whole.

Many financial assets are denominated in money terms (e.g., money itself). For these, nominal price is fixed, but not discounted or constant price. Now we consider components of change. All items considered are net - e.g. "rents" mean the excess of rents received over rents paid, "transfers" the excess of transfers to the agent over transfers from the agent (these may be < 0).

<u>Exports</u>, <u>imports</u> are the sale, purchase of goods. Thus an export is a negative investment, but leaves NW unchanged, since one's net creditor position rises as much as PA falls (ignoring transaction costs).

Net exports = exports - imports = - net imports

<u>Production</u> is the rise in quantity of a physical asset via transformation. <u>Consumption</u> is negative production. (Net) <u>interest</u> is received on financial assets and paid on liabilities. (Net) <u>rents</u> include wages, and

accrue on the difference between assets owned and assets controlled. Transfers include welfare payments, taxes, gifts, and thefts. Then, for an agent,

ΔB = (net) [exports + rents + interest + transfers]

I = (net) [production + capital gains + imports]

so

S = (net) [production + capital gains + rents + interest + transfers], since net imports cancel net exports.

Notes: adding over all agents, the following aggregates = 0: net exports, net imports, net rents, net transfers, net interest; thus, S = I = net production + capital gains, for the whole economy only. In constant dollars the capital gains term drops out.

The investment equality follows from the basic material balance identity:

Δ stock = production - consumption + net imports (in quantity terms): multiplying through by prices and adding capital gains yields investment. Problem: show that it makes no difference if transfers are of physical or financial assets.

Income concept: Letting income = S + C = I + C for the whole economy, we have discussed S and I, but C cannot be identified with consumption above, as this includes all using up of stocks. Typically one adds in only consumption in the household sector, which raises all sorts of ambiguities since there is no sharp borderline separating this sector (e.g. commuting, government services). The issue is often put in the form: is this good "final" or "intermediate", which is even more confusing since, e.g., raw inventory accumulation is part of I hence of Y. What is involved here is that people want income to represent welfare as well as net production. But (per-capita) household consumption appears intuitively to be a very poor indicator of (per-capita) welfare even in the ordinal sense. (Average lifespan does much better.)

Perfect Capital Market. We want to retain the simplicity of perfect competition in dynamic situations, hence the notion of perfection in the "capital" market (i.e. the market for dated claims on money). Assumptions

1. one universal discount rate at any time;
2. just one financial instrument: "bonds";
3. unlimited borrowing or lending allowed at the uniform rate;
4. no defaults or bad debts.

Note that one borrows by stepping up imports; borrowing in the sense of exchanging an IOU for money is meaningless here. These assumptions are inherently unrealizable, since nothing prevents agents from "living it up" to an unlimited extent. Any real world system must have credit rationing, contradicting 3; nonetheless, it may be a good first approximation for many purposes, and the alternatives are much more complicated.

Period analysis; let there be T periods; period t stretches from time t-1 to time t. Let i_t be the interest rate prevailing in period t, B_t an agent's net creditor position at time t,

W_t = agent's net [exports + transfers + rents] in period t (thought of as accruing at the end of the period)

then

$$(1)\ B_t = (1 + i_t)\, B_{t-1} + W_t \qquad t=1, \ldots, T$$

(to verify, look at $B_{t-1} \gtrless 0$ separately)

Definition: present value of the stream $[W_1, \ldots, W_T]$

$$= \frac{W_1}{1 + i_1} + \frac{W_2}{(1 + i_1)(i + i_2)} + \cdots + \frac{W_T}{(1 + i_1) \ldots (1 + i_T)}$$

Theorem: present value of $[W_1, \ldots, W_T] = \frac{B_T}{(1 + i_1) \ldots (1 + i_T)} - B_0$

This can be proved directly, but it is easier to put everything in terms of discounted dollars; the discount factor for time t is $(1 + i_1) \ldots (1 + i_t)$, so

$$B_t^* = B_t/[(1 + i_1) \ldots (1 + i_t)]$$
$$W_t^* = W_t/[(1 + i_1) \ldots (1 + i_t)]$$

the net creditor equation (1) is then simply

$$(2)\quad B_t^* = B_{t-1}^* + W_t^*$$

(proof: divide through by $(1 + i_1) \ldots (1 + i_t)$.

Present value of $[W_1, \ldots, W_T]$ is $W_1^* + \ldots + W_T^* = B_T^* - B_0^* = \frac{B_T}{(1 + i_1) \ldots (1 + i_T)} - B_0$

(proof: add equations (2) over t=1, ..., T and cancel)

The point of this result is that it enables us to formulate a lifetime budget constraint in complete analogy with the standard consumer budget constraint. Irving Fisher's idea was to combine the perfect capital market with a single constraint at death, which may be written

$B_T \geq 0$ (version I) or $NW_T \geq 0$ (version II)

i.e., you cannot die in debt (version I) or your estate must have enough assets to pay off your debts (version II). II seems more sensible, though neither is very realistic.

(Problem: formulate a better system of constraints involving collateral requirements)

Version I reads: $\sum_{t=1}^{T} W_t^* + B_0^* \geq 0$, or ,

breaking up W:

$$\text{imports} \leq \text{exports} + \text{rents} + \text{transfers} + B_0^* \quad ,$$

all terms except B_0 being the present value of the lifetime stream. This may be compared to the standard consumer budget

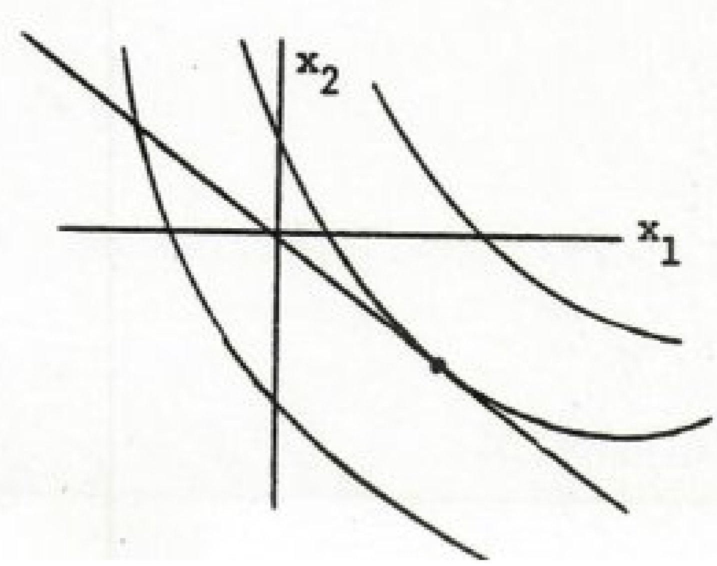
$$\text{imports} = \Sigma_j\ P_j\ x_j \leq M = \text{"income"},$$

and indicates how the term M, incorrectly labeled income, should be interpreted.

In version II we must cumulate discounted savings over time; the result is

$$\text{consumption} \leq \text{production} + \text{rents} + \text{capital gains} + \text{transfers} + NW_0$$

all terms except NW_0 being lifetime present value. In the budget constraint $\Sigma_j\ P_j\ x_j \leq M$ one usually interprets x_j as purchases or consumption of good j, lumping all sales into M. But it is just as natural to bring sales or production to the left: x_j is then <u>net</u> purchases or consumption of j and $x_j < 0$ would then be sales, etc. The utility function $U(x_1, \ldots, x_n)$ makes perfectly good sense with some $x_j < 0$. Graphically, we use all four quadrants of the plane (the budget line passes through (0,0) for the reasonable case M = 0).



Next, let goods be indexed both by form and date: x_{jt} = net purchases or consumption of good j in period t. The budget constraint then becomes

$$\Sigma_t \Sigma_j P^*_{jt} x_{jt} \leq M^*$$

Note that discounted prices must be used, because the Fisherian lifetime constraint calls for present values. M^* then is discounted lifetime rents, transfers, and initial endowment.

Next, rents can be brought over to the left-hand side.

Let S_{jt} = stock of good j held in period t

σ_{jt} = stock of good j owned in period t

r_{jt} = rental rate for good j in period t

$r^*_{jt} = r_{jt}/[(1 + i_1) \ldots (1 + i_t)]$, as usual

Then the budget constraint becomes

$$\Sigma_t \Sigma_j [P^*_{jt} x_{jt} + r^*_{jt} (S_{jt} - \sigma_{jt})] \leq M^* \quad ,$$

since $S_{jt} - \sigma_{jt}$ = net stock rented from other agents. Example: let j=1 be labor j you own yourself, so $\sigma_{1t} = 1$; in period t you rent yourself out, so $S_{1t} = 0$, and earn wage r^*_{1t}. M^* is now the present value of net transfers and initial endowments.

(The determination of M* lies in certain underdeveloped branches of economics: the economies of crime, philanthropy and political influence.)

For this set up, the appropriate utility function might be

$$U(x_{jt}, S_{jt}, j=1, \ldots, n, t=1, \ldots, T)$$

Continuous time. All the foregoing analysis may be done in continuous time instead of periods; let Δ be the length of a period. The discount factor may be written

$$(1 + i_1 \Delta)\ (1 + i_2 \Delta)\ \ldots\ (1 + i_T \Delta)$$

Now the overall time interval fixed, and let $\Delta \to 0$; the discount factor becomes

$$e^{\Sigma_t \log (1 + i_t \Delta)} = e^{\Sigma_t [i_t \Delta + 0 (\Delta^2)]} = e^{\int_0^T i(t)dt} \quad \text{in the limit.}$$

Thus discounted values are given by

$$V^*(t) = V(t) / e^{\int_0^T idt} = V(t)\ e^{-\int_0^T idt}$$

Example: if i, the instantaneous discount rate, is constant, then $e^{-\int_0^T idt} = e^{-iT}$, the familiar discount term. The present value of the continuous stream W(t), $0 \le t \le T$, is given by $\int_0^T w(t)\ e^{-\int_0^T i(\theta)\ d\theta} dt = \int_0^T w(t) e^{-it} dt$ for constant i.

Note that discount rates do for time exactly what exchange rates do for space: there is a dollar price of yen, francs, rials, etc., and there is a time-0 dollar price of time - t dollars, namely,

$$\frac{1}{(1 + i_1)\ \ldots\ (1 + i_T)} \text{ in the period analysis, } e^{-\int_0^T idt} \text{ in continuous time}$$

$$[(1 + i)^{-T},\ e^{-iT} \text{ for constant i}]$$

Problem: combine time and space to get the time-0 dollar price of time-t_0 yen; this gives the forward exchange market (under certainty).

Note there is nothing sacred about time 0; any time t_0 may be designated as the base (even a future t_0). The point is to express everything in the same t_0 dollars. In continuous time, the difference equation

$$B_t = (1 + i_t)\, B_{t-1} + W_t$$

becomes the differential equation

$$\frac{dB}{dt} = iB + W.$$

In discounted values, these read

$$B^*_t = B^*_{t-1} + W^*_t\,, \quad \frac{dB^*}{dt} = W^*\,,$$

yielding

$$B^*_T - B^*_0 = \Sigma_t\, W^*_t\,, \text{ or } = \textstyle\int_0^T W^* dt, \text{ respectively}$$

Long-term discount rate from t_1 to t_2 is defined as the constant rate yielding the same discount factor; thus in period analysis it satisfies

$$(1 + i_{long})^{t_2 - t_1 + 1} = (1 + i_{t_1})(1 + i_{t_1+1}) \ldots (1 + i_{t_2})\,,$$

in continuous time

$$i_{long}\,(t_2 - t_1) = \int_{t_1}^{t_2} i dt$$

Price rental relations: there are two basic relations: note that some prices and/or rents may not appear on the market, so must be imputed (shadow prices, quasi rents)

1. Price is the discounted value of future rents for any good:

$$P(t_0) = \int_{t_0}^{\infty} r(t)\, e^{-\int_{t_0}^{t} i(\theta)\, d\theta} dt$$

for discounted values this simplifies to $P^*(t_0) = \int_{t_0}^{\infty} r^*(t) dt$

Differentiation yields

$$ip = \frac{dp}{dt} + r \quad , \quad 0 = \frac{dp^*}{dt} + r^*$$

2. Let $F(S_1, \ldots, S_n, x_1 \ldots x_n) = 0$ be a transformation function, S_j = stock of good j, x_j = net output of good j ($x_j < 0$ for inputs being used up).

If stocks have no direct utility, a stock will be held up to the point at which the value of its marginal product equals its rental. Let F be continuously differentiable ("smooth"); then the marginal physical product of S_j in producing x_k is found by noting $\frac{\partial F}{\partial S_j} dS_j + \frac{\partial F}{\partial x_k} dx_k = 0$, so

$$MPP = \frac{dx_k}{dS_j} = - \frac{\partial F}{\partial S_j} / \frac{\partial F}{\partial x_k}$$

Multiply by P_k gives MVP, so $r_j = -P_k \frac{\partial F}{\partial S_j} / \frac{\partial F}{\partial x_k}$

This gives

$$\frac{\frac{\partial F}{\partial S_1}}{r_1} = \ldots = \frac{\frac{\partial F}{\partial S_n}}{r_n} = - \frac{\frac{\partial F}{\partial x_1}}{P_1} = - \qquad = - \ldots = - \frac{\frac{\partial F}{\partial x_k}}{P_k}$$

In more general models, stocks have direct utility (or disutility) as well as productivity (e.g. durable consumer goods); equilibrium is then the sum of these two components (convert utils to dollars by dividing by the marginal utility of money). Now consider the problem:

Max $U(x_{jt}, S_{jt}, j=1, \ldots, n, t=i, \ldots, T)$ subject to $\Sigma_j \Sigma_t (P^*_{jt} x_{jt} + r^*_{jt} S_{jt}) \leq M^*$

then $x_{kt} = f_{kt} (P^*_{jt}, r^*_{jt}, M^*)$ is the demand for x_{kt} in terms of the parameters; similarly, there is a stock demand $S_{kt} = g_{kt} (\ldots)$

<u>Theorem</u>: the demand functions are homogeneous of degree zero in P^*, r^*, M^*

<u>Proof</u>: multiplying these by a constant leaves the set of feasible options unchanged.

Furthermore, note that demand depends on nominal prices and discount rates only through discounted values; thus any changes in these leaving discounted prices invariant leaves demand invariant as well.

Example: let $M^* = 0$; let P_{jt}', P_{jt}'' be two systems of prices such that $P_{jt}'' = (1 + \lambda)^t P_{jt}'$ (P'' has an inflationary tilt compared to P'); also $r_{jt}'' = (1 + \lambda)^t r_{jt}$; also let $(1 + i_t'') = (1 + \lambda)(1 + i_t')$

Then the demand is the same under the $''$- and the $'$- system.

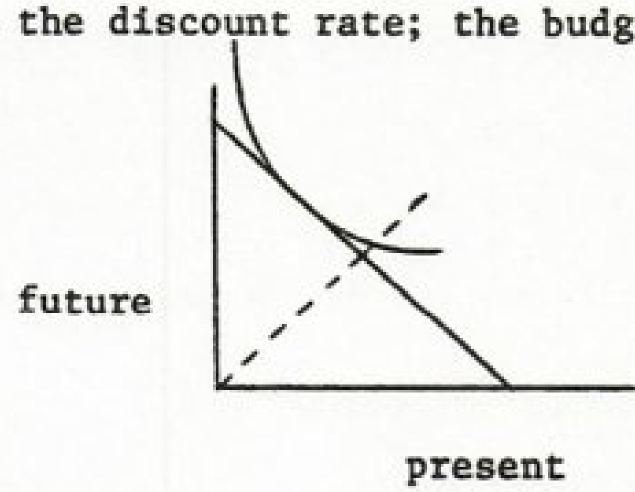
<u>Proof</u>: $$P_{jt}'^* = \frac{P_{jt}'}{(1 + i_1') \dots (1 + i_t')} = \frac{(1 + \lambda)^t P_{jt}'}{(1 + \lambda)(1 + i_1') \dots (1 + \lambda)(1 + i_t')} =$$

$$\frac{P_{jt}''}{(1 + i_1'') \dots (1 + i_t'')} = P_{jt}''^*$$

To simplify, consider the case n=1, T=2, the 2-period, 1-good case; let P_1, P_2 be prices in periods 1, 2, and i the discount rate; the budget constraint reads (ignoring rents)

$$P_1^* x_1 + P_2^* x_2 = P_1 x_1 + \frac{P_2 x_2}{1 + i} \leq M$$

Maximize $U(x_1, x_2)$



We can graph this in a familiar way:

The slope of the budget line $= - \frac{P_1}{P_2} (1 + i)$

$= -$ (1 + real rate of interest)

since P_2/P_1 = rate of inflation, there being just one good.

Definition: U has intrinsic (positive / neutral / negative) time preference if, when $x' > x''$, we have $U(x', x'') \gtreqless U(x'', x')$, respectively. (This makes no sense for two commodities at one time, since it is not invariant with measurement units.) Show: if real interest rate = 0, then

$x_1 \gtreqless x_2$ under these respective time preferences.

Profit maximization - this means: maximize present value. Why should a rational economic agent do this? Suppose there is a production sector which is utility-neutral and non-interactive with the rest of an agent's realm; then maximizing profits maximizes the range of choice of the agent under any utility function, so is part of an optimal solution:

Let y = production variables, x = other variables, P^* = discounted prices; consider the very general problem

$$\text{Max } U(x)$$

$$\text{subject to } \begin{matrix} x\varepsilon S \\ y\varepsilon T \end{matrix}, \; p^* x \leq M^* + p^* y,$$

S an arbitrary choice set not dependent on y; the optimal solution clearly requires that y maximize $p^* y$ subject to the technology constraint $y \, \varepsilon \, T$.

There is also the "natural selection" argument that economic power (measured by Net Worth) gradually flows toward those agents who are most strongly motivated to acquire it.

Technology - a time extended technical process may be described by a $2N\theta$ - dimensional vector $(S_{jt}, x_{jt}, j = 1, \ldots N, t = 1, \ldots \theta)$

S_{jt} = stock of good j at time t,

x_{jt} = net output of good j at time t

The technology set T is the subset of $2N\theta$ - dimensional space of all technically feasible processes. The efficiency frontier is the subset of T such that no output can rise, or input or stock fall, without an opposite change in some other component.

T may be built up from more elementary production processes e.g. in period analysis, an initial bundle of stock at time t-1 gets transformed into a final bundle at time t (which, after additions and withdrawals becomes the initial bundle for the next period). In continuous time, technology may be taken as a 2N dimensional set of vectors $(S_1, \ldots, S_N, x_1, \ldots, x_N)$ S_j = stock of good j, x_j = net output of good j.

"Factors" are those goods which transform slowly in production (catalysts), "materials" transform rapidly. Depreciation, product improvement, learning, fatigue, etc. may be represented by distinguishing different quantities of a good as different goods-e.g., a new machine disappears and a worn machine appears.

There is no fundamental distinction between production and household activities. Much production occurs in the home - cooking, cleaning, education, nutrition. Reproduction is a form of production. Conversely, most business activities have a welfare aspect ("working conditions"), and we may write $W(S, \ldots, S_N, x_1, \ldots, x_N)$ as the welfare generated in using the stocks S to yield input output pattern x, W being experienced by the persons participating in the process and perhaps by others.