MEASURE-THEORETIC LOCATION THEORY

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This paper is mainly an exposition of some basic concepts in my recent book, Economics of Space and Time: The Measure-Theoretic Foundations of Social Science.[1] Toward the end I shall also outline some further results I have obtained, and try to forecast the future development of the measure-theoretic approach, and its significance for regional science and the social sciences generally.

The Transportation Problem

Consider first the well-known transportation problem of linear programming, which is to minimize

$$\sum_{i=1}^{m} \sum_{j=1}^{n} t_{ij} \; x_{ij} \qquad (1)$$

over $x_{ij} \geq 0$ subject to the constraints

$$\sum_{j=1}^{n} x_{ij} \leq c_i \;, \qquad (2)$$

i=1, ..., m, and

$$\sum_{i=1}^{m} x_{ij} \geq r_j \;, \qquad (3)$$

j=1, ..., n. In the most straightforward interpretation, a commodity is

[1] Iowa State University Press, Ames, Iowa, 1977 (700 pp.). This will be referred to as EST.

to be shipped from m sources with capacities $c_1, \ldots, c_m$, to n sinks with requirements $r_1, \ldots, r_n$. t_{ij} and x_{ij} are the unit costs and shipments from source i to sink j.

Think of grain shipments from farms to markets. We might let i run over all the individual farms in the economy. It is just as natural, however, to think of sources as being continuously distributed over the landscape, so that each point of the earth's surface is a potential source. Any one point has a capacity of zero, however, so capacities must be represented in a different way. For any region E, let $\mu(E)$ be its grain capacity. μ is then a function that assigns (non-negative) numbers to regions. Furthermore, μ is additive: if E and F are disjoint regions, the capacity of the combined region $E \cup F$ is clearly the sum of the two individual capacities:

$$\mu(E \cup F) = \mu(E) + \mu(F) \ . \tag{4}$$

In short, μ is a measure on the space of sources.[2] Intuitively, a measure is a mass distribution over a space, assigning "weights" to subsets of that space. Here the space is literally physical space, the "weight" of a region being its grain capacity.

Note that the original transportation-problem capacity assignment is a special case of a measure, the discrete distribution assigning weight c_i to the singleton set, point i.

In a similar manner we may represent sink requirements by a measure: $\nu(E)$ is the grain requirement for region E. Now consider the shipments.

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[2] In the interest of brevity I ignore all questions of measurability and countable additivity. Note that regions are thought of a point sets, so that $E \cup F$ is the set-theoretic union.

The flow x_{ij} assigns a number to each ordered pair (source point, sink point). Thus, if A and B are the source and sink spaces, the flow is defined on the product space $A \times B$ -- in fact as a measure λ over this space. For regions $E \subset A$, $F \subset B$, $\lambda(E \times F)$ is the mass of grain shipped from region E to region F. One verifies that λ is indeed additive.[3] Note that $\lambda(E \times B)$ = total shipments out of region E. The capacity constraint (2) now reads:

$$\lambda(E \times B) \leq \mu(E) \ , \tag{5}$$

for all $E \subset A$. That is, shipments out of region E cannot exceed the capacity of E. Similarly, the requirement constraint (3) reads:

$$\lambda(A \times F) \geq \nu(F) \ , \tag{6}$$

all $F \subset B$. That is, total shipments into region F must meet the requirements of that region.

Finally, letting $t(a,b)$ be the unit cost of shipping from point $a \in A$ to point $b \in B$, the sum (1) becomes an integral[4]:

$$\int_{A \times B} t \, d\lambda \tag{7}$$

The measure-theoretic transportation problem, then, is to find a measure λ on $A \times B$ minimizing (7) subject to the constraints (5) and (6).

(Note that t remains a "point" function, while c, r, and x become

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[3]This is clear if $A \cap B = \emptyset$. If they overlap, interpret λ as gross shipments, including shipments of a region to itself.

[4]The integral is formally identical to the expection of the "random variable" t with respect to the "distribution" λ in probability theory, except that $\lambda(A \times B)$ need not equal one.

"set" functions, in the transition from (1), (2), (3) to (5), (6), (7). The reason is that unit cost is an "intensive" magnitude, while capacity, requirement and flow are "extensive" magnitudes).

The Problem of Description

We shall follow up on the transportation problem shortly. But first consider: are there any other real-world situations that seem naturally describable in terms of measures? The answer is that measures have universal descriptive power. I will indicate briefly how the apparatus of concepts is built up.[5] We start with three underlying sets -- physical space, S, the set of locations; time, T, the set of instants; and resource space, R, the set of possible quality types. Think of each point of R as being a detailed description of a type of entity -- e.g. the physico-chemical properties of a substance, or the physical and mental state of an individual. Consider production in an economy. A complete description would state how much of what is produced where and when. This may be represented by a measure μ over the space $R \times S \times T$, namely, $\mu(E \times F \times G)$ equals the mass of resource-types E produced in region F in period G, for all $E \subset R$, $F \subset S$, $G \subset T$. If one is interested only in certain resource types or certain regions or periods, simply restrict μ to the appropriate subspace -- e.g. coal mining in Appalachia in the 1960's. If one is interested in the space-time pattern of total value added, marginalize over R, that is, consider the measure ν on $S \times T$ given by

$$\nu(H) = \mu(R \times H) , \qquad (8)$$

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[5]For full details see EST, Chapter 2.

$H \subset S \times T$, where μ and ν are now appropriately expressed in real dollars. (Changes of measurement units are easily handled; "mass" may be in terms of numbers of units, dollars, hectares, board-feet, etc., as befits the entities in question.)

More generally, let a "history" be a mapping from an interval of T to $R \times S$, yielding a description of a hypothetical entity over time. The actual world may be described as a measure over Ω, the set of all histories, $\mu(E)$ being the mass embodied in the histories of $E \subset \Omega$.[6] Marginalizing μ in various ways gives the specific measures that constitute the bulk of compilations such as the <u>Statistical Abstract of the US</u>: production, consumption, exports, inventories, births, deaths, marriages, migration, bankruptcies, etc. These are all "extensive" magnitudes. Of the remaining data, the great bulk are "intensive" magnitudes, and are technically <u>densities</u> from the measure-theoretic point of view. Prices, for example, are the densities of money values with respect to physical quantities, both measures, and we have the relation

$$\nu(E) = \int_E p \, d \, \mu \tag{9}$$

where ν, μ are the value and physical measures over, say, $R \times S \times T$, and $p(r, s, t)$ is the price of resource type r at location s at time t. (Note that prices are point functions, as are all densities. $E \subset R \times S \times T$). Similarly, per-capita incomes are the densities of income measure with respect to population measure, price indexes are the densities of money measures with respect to real measures, interest factors are the densities

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[6]Note the formal resemblance to a stochastic process, "history" corresponding to a sample path or realization.

densities of discounted values with respect to money values, etc.

Measure Theory as a Natural Language

Even if we grant that the world may be described in terms of measures, there still remains the question, what is the point of it? Consider some practical issues faced by a regional scientist.

(a) Describe the distribution of population at time t. A complete description would list everybody in the society and give the location of each. This is neither feasible nor desirable -- it would just be confusing. Instead, what one wants is to specify a relatively small number of parameters that catch the essential features of the distribution. One approach -- universal in censuses -- is to partition the landscape into counties, tracts, etc., and count the number of people in each unit. It is important to note that this is not the only way, and not necessarily the best way, of doing things. For city populations, Colin Clark, E. S. Mills and others have shown that population densities are well-described by a circular exponential distribution

$$\delta(x) = D_0 e^{-\gamma|x-x_0|}, \tag{10}$$

where x_0 locates the center of the city, $|x-x_0|$ is Euclidean distance, and D_0, γ are parameters.[7] The urban population distribution of a society may then be expressed as a summation of densities of the form (10), one for each city. Still other approaches are possible -- e.g., via the

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[7]Colin Clark, "Urban Population Densities", J. Royal Stat. Soc., A114 (1957) 490-96.

E. S. Mills, Urban Economics, (Scott, Foresman, 1972) Chapter 6, and paper presented at ASSA Convention, Chicago, August 1978.

potentials of J. Q. Stewart[8], via central place models, as a non-homogeneous poisson process, etc. To get an overarching view of these possibilities, one must start with a formulation that allows a wide variety of possible measures.

(b) Find the distribution of an industry that minimizes the total cost of meeting a specified pattern of demand. This typical location problem may be formulated as follows: let $\nu(E)$ = mass to be delivered to region E; find $\mu(E)$ = production in region E, $E \subset S$, to minimize costs of production plus transportation. Scale economies or zoning restrictions may preclude $\mu = \nu$. Here one should not prejudge the form of μ: it may be discrete, continuous or mixed. To take a striking example: let ν be uniform on the plane, let transport cost be proportional to Euclidean ton-miles, and let the cost of producing x at a point be given by

$$ax^2 - bx^{3/2} + cx \tag{11}$$

Then, for certain parameter values a, b, c yielding a classical U-shaped average cost curve, the optimal distribution μ is neither discrete nor continuous.[9]

(c) These examples do not clinch the issue, however, since at worst they seem to require looking at a variety of two-dimensional distributions. There remains a big gap between this and the abstract measure theory used in, e.g., the transportation problem above.

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[8] J. Q. Stewart, "Empirical mathematical rules concerning the distribution and equilibrium of population," Geog. Rev. 37 (July 1947) 461-485.

[9] A. M. Faden, "Inefficiency of the regular hexagon in industrial location," Geog. Analysis 1 (Oct. 1969) 321-328, cf. EST, 667f. Similar mixed results are to be found in E. M. Hoover, Location Theory and the Shoe and Leather Industries (Harvard Univ. Pr., 1937).

Consider the situation in probability theory. This itself is a branch of measure theory, but the full power of the latter is usually not needed in ordinary problems of finite dimension. Measure theory comes into its own with stochastic processes, involving an infinity of random variables, and allied problems of convergence and representation. Are there similar real-world complexities? Yes, there are. First, resource space R is complex in ways we have scarcely begun to think about. Second, the space of histories Ω already has a structure similar to a stochastic process, as mentioned above. Third, consider the introduction of uncertainty. The world is describable by a mass distribution, but we don't know which one. Thus we have a probability distribution over space of mass distributions, and must deal with random measures.[10] I will not discuss these three points further. The next, however, is more central to my argument.

Consider the concept of land use. A land use at a site may be described by the time-pattern of inputs and outputs, i.e., by a pair of measures over $R \times T$: $\mu^{out}(E \times F)$ = mass of resource types in E produced at the site in period F, for $E \subset R$, $F \subset T$, and similarly for μ^{in}. Assuming constant returns to scale in all outputs and inputs, including land, μ^{out} and μ^{in} are interpreted as per hectare, so that $x\mu^{out}$, $x\mu^{in}$ are obtained from x hectares. Let Q be the set of all feasible land uses. Now consider the concept of an assignment of land uses. This is a measure λ over the set $S \times Q$, with the interpretation:

$\lambda(E \times F)$ = total hectares in region E occupied by land uses in F, for all $E \subset S$, $F \subset Q$.

[10] See EST, 2.8.

Note that λ is a "second-order" measure, since Q itself is a space of measures. The construction seems necessary to capture the concept of land-use assignment.

Now consider the transportation problem (5), (6), (7) again. The formulation is abstract, and the problem has a theory which parallels and extends the theory of the ordinary transportation problem (concerning existence of feasible and optimal solutions, duality and potentials).[11] The ordinary transportation problem also has other interpretations, one of which is the optimal assignment of resources to jobs--or of land to land uses. Thus consider the problem

$$\text{Minimize } \int_{S \times Q} t \, d\lambda \qquad \text{over measures } \lambda \tag{12}$$

$$\text{subject to } \lambda(E \times Q) \leq \mu(E) \text{ , all } E \subset S \tag{13}$$

$$\text{and } \lambda(S \times F) \geq \nu(F) \text{ , all } F \subset Q \tag{14}$$

This is the same as (5), (6), (7) except for notation. Interpret $\mu(E)$ as the area of region E. $\lambda(E \times Q)$ is total hectares in region E assigned to all land uses together. Thus (13) is an areal capacity constraint. Interpret $\nu(F)$ as an allotment of hectares to land uses F. $\lambda(S \times F)$ is the total area assigned to F by λ. Thus (14) requires that allotments be met.[12] Finally, t(s, q) is the cost incurred per hectare

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[11] EST, 7.1 - 7.5. A less advanced version appears in Faden, "the abstract transportation problem", Papers in Quantitative Economics (Univ. Pr. of Kansas, 1971) Vol. 2, 147-175.

[12] E.g., that at least 5 hectares be devoted to growing turnips. To test whether an actual system optimizes (12) subject to (13) and (14), set ν equal to the actual land use allotment in that system.

when land use q is assigned to site s, and (12) is the total cost incurred by assignment λ.

We now show that the classical von Thünen model (in an optimizing sense) can be represented as a transportation problem (12), (13), (14). This requires only that the cost function t(s, q) be further specified. Now in the classical Thünen system there is a central point of attraction (e.g. market center, central business district, etc., depending on the interpretation). Call this point s_0, the nucleus. Let h(s) be the distance from site s to s_0. A land use q located at s incurs a transport cost in that inputs and outputs must be shipped from and to s_0. Let w(q) be present value of costs generated by q per hectare per kilometer (ideal "weight" of q). Then

$$t\,(s,\, q) = h(s)\; w(q) \tag{15}$$

specifies t. Actually, all of the basic Thünen results fall out from a more general version of t, namely,

$$t(s,\, q) = f[h(s),\, w(q)] \tag{16}$$

where f is any function having positive cross-differences [$\Delta x\ \Delta y\ f(x, y) > 0$]. The problem of minimizing (12), subject to (13) and (14), where t has the form (16), is called the allotment-assignment problem.[13]

Before giving the results of this reduction of Thünen to allotment-assignment, it is striking to note that the reducibility of Thünen to the

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[13] EST, 8.5. h, w, and μ need not be physical distances, weights, or areas, respectively, but may all be "ideal". Thus the Thünen model represents an enormous variety of situations and geometrical forms. EST, Chapter 8 passim.

transportation problem went unrecognized in the literature.[14] I stress this point, because it underlines where the measure-theoretic approach (specifically, the measure-theoretic transportation problem) comes into its own. The reduction is fairly obvious once made, but to make it one must overcome the psychological barrier of assimilating the "continuous" landscape of the Thünen model to the "discrete" ordinary transportation problem. The barrier breaks automatically in going to measures, hence the virtue of generality. Even more, the abstract character of the measure-theoretic approach justifies our taking the very messy space Q of all land uses--certainly ∞-dimensional in realistic situations--as our sink space B. Thus our results embrace the full range of urban and rural land-uses, with their variety of intensities, timings and successions.

From the allotment-assignment problem one may deduce the full qualitative charactor of Thünen systems: the inverse relation between the "weight" of land uses and (ideal) distances from the nucleus, and land values as a decreasing convex function of ideal distances. (One of the dual variables--the "potentials" of the allotment-assignment problem--turns out to be land-value density.)

Brief Survey of Other Results

We have no time to go into the other models in EST in as much detail as with Thünen above, but instead outline a few results to indicate the scope, generality and unifying power of the measure-theoretic approach.

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[14]The closest approach, in effect, was to recognize the "one-sided" problem omitting constraint (14), which just yields an uninteresting local result. See B. H. Stevens, "location theory and programming models," Reg. Sci. Assn. Papers 21 (1968) 31-34. M. Beckmann and T. Marschak, "an activity analysis approach to location theory" Kyklos 8 (1955) 128.

The bulk of mainstream location theory from the works of Launhardt, Weber, Palander, Hoover, Lösch, Isard, Beckmann, and Alonso is covered. Here are some examples.

(a) Market areas. We noted that the Thünen model falls out as a special case of the transportation problem. By specializing the transportation problem in a different way, we get the market area model--namely, by letting either the source space A or sink space B be countable. The familiar geometric pattern of each plant supplying (or being supplied by) its local hinterland emerges, though the results are valid for general "non-Euclidean" transport cost functions. (EST 9.5)

Both the market area and Thünen models are also formulated in equilibrium rather than optimizing terms, and, assuming competition, the actual spatial price fields of the former are the same as the shadow prices of the latter which emerge from the dual transportation problem.

(b) Hierarchies, central places, metropolitan dominance, and geopolitics. By letting transport costs arise from a branching tree road structure, the Thünen model attains many of the qualitative features of a metropolitan region, with satellite towns following the central place pattern in terms of the land uses found in them. Applying the Thünen model to the world as a whole (the nucleus being the "industrial heartland" around the North Atlantic), one can explain certain gross geographical features--the pattern of income densities, the fact that the interiors of continents tend to be less densely populated than their fringes, the density of settlement along navigable rivers, etc. (EST 8.8 - 8.9)

(c) Transhipment. Modify the transportation problem by letting source and sink spaces be the same (A = B), and replacing the constraints (5) and

(6) on shipping measure λ by the single system

$$\lambda(A \times E) - \lambda(E \times A) \geq \mu(E) , \tag{17}$$

all $E \subset A$. Now $\lambda(A \times E)$ is gross shipments into region E, $\lambda(E \times A)$ is gross shipments out of region E, so the left side of (17) is <u>net</u> shipments, and (17) demands that this meet the net requirements $\mu(E)$ of region E.[15] The objective function remains (7), with A = B. This is the measure-theoretic transhipment problem.[16]

(d) <u>Interindustry location</u>. Suppose unit mass is to shipped from origin distribution μ_1 to destination distribution μ_2 in space S [$\mu_1(S) = \mu_2(S) = 1$]. We have the transhipment constraint

$$\lambda(S \times E) - \lambda(E \times S) \geq \mu_2(E) - \mu_1(E) , \tag{18}$$

all regions E, and are to minimize

$$\int_{S \times S} t \, d \, \lambda \tag{19}$$

subject to (18), where t is the transport-cost metric. Write the minimal shipping cost as $T(\mu_1, \mu_2)$. Now suppose there are n industries, and mass W_{ij} must be shipped from industry i to industry j. The total cost

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[15] Here μ is a <u>signed</u> measure, which may take on negative values. (Think of a distribution of electric charges rather than a distribution of mass.)

[16] EST, 7.6 - 7.11. The exposition in EST is incomplete. I have since obtained results for transhipment potentials parallel to those for transportation potentials (unpublished).

incurred is then

$$\sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij} \, T(\mu_i, \mu_j) \tag{20}$$

Problem: Given the location of two industries, μ_1 and μ_2, given metric t and the matrix W_{ij}, find the location of the remaining n-2 industries to minimize (20).

Theorem: There is an optimal solution in which each μ_i coincides either with μ_1 or with μ_2.

This result holds for arbitrary measures in an arbitrary metric space, and is one of a number of "coincidence" theorems which may cast some light on the phenomenon of spatial agglomeration (EST 9.3).

(e) The real-estate market. Consider an economic agent's preferences concerning the control of real-estate. In a simple form, the possible options may be thought of as pairs (E, x), E being a region and x the cost of acquiring E. E is perhaps best thought of as "4-dimensional"-- this collection of lots for these time intervals - i.e., $E \subset S \times T$. Letting μ be the rental measure over space-time, each agent i chooses his region E_i to maximize $U_i[E_i, \mu(E_i)]$, and equilibrium obtains when the resulting E_i's partition the subspace of $S \times T$ that is on the market. The problem is to characterize the equilibrium and find when it exists. (EST 6.4 - 6.8.)

(f) Crime distribution. Consider the spatial distribution of police, criminals, and potential victims (or wealth). Criminals move to where the wealth or victims are, police also move there to deter crimes, and victims or wealth may move away if crime incidence is significant. A plausible crime density function, for example, is

$$c\ v\ e^{-p} \tag{21}$$

where c, v, p are the densities of criminals, victims, and police (per hectare). We have a game-theoretic situation with two (or three) players. If only c and p adjust, for example, it may be shown that the crime function (21) yields a two-regime solution: There is a critical $\bar{v}$ such that, in the region $\{s \mid v(s) < \bar{v}\}$ there are no police and no criminals, while in the complementary region criminals are uniformly distributed, while police distribute themselves to make $v\ e^{-p}$ constant. (The police objective is to minimize total crimes.) Oddly, crimes per victim are highest just above $\bar{v}$. Also, an increase in total police leads to a lowering of $\bar{v}$, so that the regime of crime actually spreads (though total crime falls). (EST, 5.8.)

(g) Löschian industries. Adding benefit or revenue functions as determinants of demand, and production cost functions to the market area set up yields a variety of Löschian one-industry models, under various competitive, monopolistic or publicly operated conditions (EST 9.6). The problem with variable plant locations is still unsolved.[17]

(h) Network flow models may be put in measure-theoretic form. There are generalizations of the "min-cut-max-flow" and allied theorems (unpublished).[18]

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[17]The common statement that plants should arrange themselves to have regular hexagonal market areas is wrong, even under simple uniform conditions. See note 9 above.

[18]L. R. Ford, Jr., and D. R. Fulkerson, Flows in Networks (Princeton Univ. Pr., 1962), survey the basic "discrete" theory.

Conclusion

These examples illustrate the sense in which measure theory is the natural language for location theory, just as differential equations are the natural language for most of physics, or (significantly) measure theory itself is again the natural language for probability and statistics, the study of uncertainty.

Actually, we may distinguish two senses in which a language may be "natural" for a given discipline--its expressive power vs. its algorithmic or problem-solving ability. Now measure theory is unsurpassed in its expressive power--its ability to deal with complicated sets such as the space of histories or land-uses, with "non-Euclidean" areas, distances, and weights, with the discrete and the continuous, etc. It provides us for the first time with the ability to come to grips with what is involved in these complex phenomena. And it allows us to grasp the underlying unity of apparently diverse location models.

On the other hand, measure theory is not particularly algorithmic, in contrast to parts of the calculus, or algebra, or computer languages. Thus it must be supplemented by practical techniques, and these will probably resemble present-day practices in regional science for some time to come. Is anything to be gained from measure theory at this level? First, it supplies a wider variety of models for which algorithms are to be found. Second, it may be needed to verify the effectiveness of proposed algorithms. Third, it provides insights suggesting new and better algorithms.

For example, is it always best to approach regional problems by partitioning the landscape into a small number of regions, partitioning industries, goods, and people into small number of types, etc., and then working with aggregated stocks and flows within these partition elements? In probabilistic terms this is equivalent to approximating arbitrary distributions by discrete distributions only. Might it not be better for some purposes to approximate population distribution, for example, by a mixture of circular exponentials, as suggested by the Colin Clark - E. S. Mills results?[19] Effective algorithms require approximation by a space with a small number of parameters to be searched, but there are a great variety of ways to set this up. One needs the birds-eye view of the measure-theoretic approach to make an intelligent search for the algorithms themselves.

Prospectus

I will conclude with some speculations about the future of the measure-theoretic approach, based partly on my own unfinished research.

First, a much wider collection of theoretical models remain to be developed. Almost none of the models in EST is truly dynamic in the sense that the future unrolls from laws applied to the past (as in differential equations or markov processes). Almost none involves uncertainty. The comparative advantage of the measure-theoretic approach can only grow as these richer dimensions are introduced. Uncertainty requires the consideration of "two-layered" theories at least, with random measures, as discussed above, hence has all the complexity of stochastic processes and then some. Think of the problems involved with dynamic interindustry linkages, agglomeration effects, the differential

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[19] See note 7 above.

distribution of information over space and time. We need all the expressive power we can get!

Next, the foundations of the measure-theoretic approach itself need clarification. For example, how does one translate between description in terms of everyday language and measure-theoretic descriptions. Clearly people do not literally run around with measures in their minds, no more than with probability distributions. Resource space R has a structure which requires elucidation--e.g. how some resource types can be parts of others, or how "sense-data" variables relate to "thing" variables. What are the measurement processes by which one arrives at measures?

Next, consider the position of location theory among the sciences. I was attracted to it out of the conviction that the deep study of space (and time) holds the key to understanding the world in general. Thus location theory should be pivotal in the hierarchy of the sciences, a position it hardly occupies at present. If measure theory is its natural language, the world picture that it suggests should be fruitful for attacking some of our more fundamental persisting problems. I will give two such applications. (Again, these ideas are definitely half-baked as of now, though more developed than the following casual remarks might indicate.)

The foundations of probability theory lie at the heart of epistemology, which asks, "how is science, or everyday knowledge, possible?" or "what inferences are we justified in drawing from our own experiences?" The various attempts at an industive logic--by Keynes, Carnap, and Hintikka--seem grossly inadequate. If nothing else, they deal only with discrete random variables. My suggestion is that, as a preliminary, one needs to understand the space over which these probability distributions are to range. The measure-theoretic picture is richer than seems to have been envisioned by these investigators, and states that one should look at the set of possible measures over the world of histories. The kinds of random variables suggested by this picture are richer than relative frequencies--they are random measures, and indicate that questions such as the distribution of mass in a space-time region, or the distribution of longevity of entities of a certain type, are the kind that should be tackled first.

Again, even if we has a satisfactory inductive logic, people would deviate from it owing to the cost of calculation. We work our way through an uncertain measure-theoretic world by filtering out some information and distorting much of the rest (e.g. by "point estimating"--i.e., replacing a non-degenerate by a degenerate distribution). The entire theory of statistical inference can be looked at as a trade-off between the costs of complexity and the costs of inaccuracy, the pole of perfect accuracy being "bayesian" inference with respect to the "true" prior of inductive logic.[20] The role of hypothesis in science comes into its own again on this approach.

Finally, the basic idea of the measure-theoretic picture--that the world is matter being redistributed among different locations and qualities (including mental qualities)--suggests that one look at the rates of redistribution and their causes. This brings us to models of the natural selection type, in which one examines differential survival

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[20] A. Faden and G. Rausser, "Econometric policy model construction: the post-bayesian approach," Annals Econ Social Meas. 5 (1976), 349-362.

not only of organisms, but of ideas, ideologies and institutions. But to go into this would involve another long paper in itself.