THE LOGIC OF PRICE AND PRODUCTIVITY INDEX NUMBERS

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SUMMARY

1. The relation between Marshallian surpluses, their generalizations, and measures of aggregate income and wealth are examined. The conclusion is that the assumption of 1st degree homogeneity of the wealth function is necessary and sufficient to justify the usual measures of aggregate income and wealth, but only if the price index assumes the special form which we call the Universal Deflator, or simply The Index. This must exist only on a world-wide basis, must include human capital, and must be weighted by stocks rather than flows.

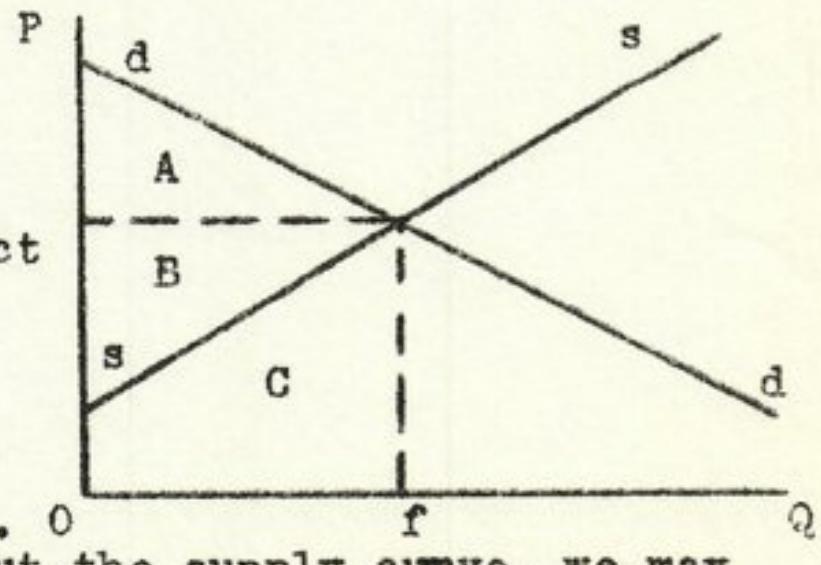
We argue that, within the framework of our assumptions, the Universal Deflator is the only price index with which economic theory need concern itself, and that all other price indices are illegitimate except as approximations to this one. We then discuss some biases which are likely to arise in practice, and an example of misapplied regional index numbers.

2. What happens when the assumptions underlying The Index break down is discussed next. In particular we analyse the concept of external economies, and, in connection with this, the mysterious increasing efficiency of resources which several investigators have noticed.

3. Three Appendices deal with the concept of a wealth inventory, with the measurement of regional price indices, and with the complications introduced by lags and frictions, respectively.

SURPLUS, WEALTH, AND THE UNIVERSAL DEFLATOR

There are two methods commonly advocated for measuring value. These may be conveniently illustrated by means of the familiar supply-demand diagram. The first, which is supposed to be more perfect in theory but harder to measure, may be called the surplus method, and takes total gains from trade to be the sum of buyers' surplus A and sellers surplus B (we neglect income effects for the time being).



Even more simply, if we forget about the supply curve, we may interpret the diagram as denoting the price of a good as a function of the stock of it in existence. Then if Of is the quantity in existence, the surplus method takes the total value of the stock to the economy to be the sum A plus B plus C (neglecting "wealth effects").

The second, which may be called the national income method, simply measures value by the product of price and quantity: in the diagram, B plus C.

These measures become more interesting when one attempts to aggregate them for an entire economy. Here the surplus method runs into very

hot water indeed. First of all the adding up of separate surpluses into a single grand surplus is illegitimate: a reclassification of commodities would lead to a completely different grand value.[1]

[1] Hotelling makes this mistake in his pioneering article applying surplus analysis to welfare problems "The General Welfare in Relation to Railway and Public Utility Rates" Econometrica 1938
Taxation and

Second, income or wealth effects, which may be slurred over for a single unimportant article, become of crucial importance. When the measurement of surplus for even a single article presents such difficulties (because of our lack of knowledge of the shapes of the demand and supply schedules) it would seem that the attempt to aggregate for the whole economy must be hopeless. As we shall see, however, such a judgment would be a bit too hasty.[2]

[2] Marshall, a strong advocate of the surplus method, commented: "...the task of adding together the total utilities of all commodities, so as to obtain the aggregate of the total utility of all wealth, is beyond the range of any but the most elaborate mathematical formulae. An attempt to treat it by them some years ago convinced the present writer that even if the task be theoretically feasible, the result would be encumbered by so many hypotheses as to be practically useless." (Principles of Economics 8th ed 131n)

The national income method blithely cuts through the aggregation problem by adding up all the separate price-quantity products, avoiding double counting, and making numerous quasi-arbitrary adjustments, the legitimacy of many of which is still in dispute. The justification for adding appears to be expediency: in the world the only figures which one has access to are prices and quantities, and given these the general national income approach is about the only one which can be used to arrive at any figure at all. One hopes that the figure arrived at, when suitably deflated, will vaguely conform to the true, but inaccessible, generalized surplus measure.[3]

[3] cf. the elaborate discussion in Pigou Economics of Welfare PtI

Analysis

Suppose an economy consists of a group of entities A, B, C,..., each being a person, a machine, a piece of land, etc. Suppose further there is a way of measuring the total social wealth W(A, B, C,...) and also the total wealth if A did not exist: W(B, C,...). The difference between these two we call the conditional value of A given B, C,...: W(A/ B, C,...). From the remainder B, C,... we can separate off B in the same way, getting the conditional value of B given C,...: W(B/ C,...). This splitting may be continued until all the entities in the economy have been exhausted, giving a breakdown

W(A, B, C,...) equals W(A/ B, C,...) plus W(B/ C,...) plus W(C/...)
plus ...

(Note the perfect analogy with conditional probability, if only products are substituted for sums).

This breakdown is exactly the logic behind the surplus approach. We assume that the value of an entity is correctly measured by its (deflated) price: that is, we assume there are no net external effects. Now we select all the entities of a given type. We take one unit of these and measure its value by its price; we then take another and measure its value by the price it would have if the first unit we took no longer existed; we take still another and measure the price it would have if the first two units no longer existed; and so on until all entities of this type have been accounted for. As the entities become scarcer these hypothetical prices will presumably rise. The total value obtained is exactly the area A plus B plus C in the diagram on page 1, interpreting it now in the stock and not in the flow sense. (We stillare neglecting wealth effects)

It is now clear how to generalize the surplus measure. After the first type of entity has been fully accounted for, we select all the entities of a second type and continue the process: the first unit is valued at the price it would have if no entities of the first type existed; the second is valued at the price it would have if no entities of the first type existed and also the first unit now measured did not exist; and so on through all entities of this second type. And one goes on in an analogous manner through all the remaining types.

But what about the wealth effect? As we break down the economy step by step into its component entities the price level will also vary, the prices of all entities adjusting to the new relative scarcities. To keep values comparable it is therefore necessary to adjust the deflator as one goes along. The wealth effect is essentially a problem of the price index, and, as we shall see, han be allowed for handily by using the Universal Deflator.

The method outlined above still appears to be impossible to carry out in practice. Suppose instead we attack the aggregate wealth function directly. In particular is there any basis for believing that the expedient national income method (national wealth in this case) is any good approximation to the true wealth function built up by surpluses. That is, we desire to know whether

1) $$W(x_1, x_2, x_3, \ldots) \text{ equals } \frac{S_i\ x_i p_i}{U}$$

where the x_i now stand for quantities of the various types of stocks in existence, the p_i for their prices, S indicates summation, and U is the general price index.

Since we are assuming no net external effects, p_i/U measures the marginal contribution to wealth of a unit of type i. But if this is so the equation above is Euler's theorem for 1st degree homogeneous functions. Can we then conclude that W is 1st degree homogeneous? Yes, since this converse of Euler's theorem is also true.[b]

[b] Widder Advanced Calculus

We therefore conclude that, with no net external effects, a necessary and sufficient condition for the national wealth method to truly estimate total wealth is for the wealth function to be 1st degree homogeneous. We discuss below the conditions under which

this is likely to occur.

The general price index number $\underline{U}$ is by no means arbitrary, and its form may be determined as follows:

Differentiating the equation above with respect to time yields

2) $\frac{dW}{dt}$ equals $S_i \frac{p_i dx_i}{U\ dt}$ plus $S_i\ x_i \frac{d(p_i/U)}{dt}$

But also

3) $\frac{dW}{dt}$ equals $S_i \frac{\partial W}{\partial x_i} \frac{dx_i}{dt}$ equals $S_i \frac{p_i dx_i}{U\ dt}$

Therefore

4) $S_i\ x_i \frac{d(p_i/U)}{dt}$ equals zero

This differential equation may be solved for $\underline{U}$ to yield

5) $\frac{U(t'')}{U(t')}$ equals $\exp \int_{t'}^{t''} \frac{S_i\ x_i \frac{dp_i}{dt}}{S_i\ x_i p_i}\ dt$

This is the Divisia-Roy index number.[5] One may verify that it

[5] Davis Theory of Econometrics

satisfies all of Fisher's desiderata for index numbers. It does so by depending not on the values of prices and quantities at the end-points of a time interval, but on the entire paths of prices and quantities over the interval. If price and quantity data at fairly close intervals are available the Divisia-Roy index number may be approximated by a chained-link index, the more links the better. Any conventional index may be used for the links since they all converge as the number of links increases.

Implications

Measurement of wealth by the summation of price-quantity products requires that we locate a 1st degree homogeneous wealth function and use the right general price index deflator. The first requirement means essentially that the economy must be comprehensive and closed. For example, is it correct to exclude human beings from the stock of wealth, as is almost always done? If the stocks of all items of non-human wealth doubled and the stocks of human beings also doubled, then total wealth will have doubled , in the absense of economies of scale. But if only the former occurred, then the value of non-human wealth will have less than doubled, because of its growing abundance relative to human beings. (One may, if one likes, express this by saying that the marginal utility of money has declined; then this approach provides a method for measuring that decline). Again, suppose all the wealth in one region of an interregional trading system doubles in quantity; will it have doubled in value? No, because the terms of trade would move against that region. In short only a closed economy can hope to have a

1st degree homogeneous wealth function. Since there are no closed economies short of the entire world, the entire world must be considered. (see Appendix I for problems arising from the measurement of wealth on a comparable international basis).

This may appear to be a tall order, but once the general international-human-capital-price index is established, a great compensating advantage appears: we may dispense with all index numbers other than U! U is the factor making our numeraire measuring rod comparable with itself over time, and with this one adjustment all comparisons may be made in undeflated terms. (The existence of lags and frictions make some qualifications to this bald statement. see Appendix III). One may dispense with the medley of wholesale-price indices, cost-of-living indices, regional price indices, etc. In fact, we will make a stronger statement. In the sense in which economic agents are supposed to adjust to real values rather than money values (no "money illusion") it is erroneous to deflate by any index except U to go from money to real terms. In this sense we may refer to U as the Universal Deflator, or simply The Index.

An example to illustrate this last statement. Consider two trading regions, one of which has both wages and the cost-of- living higher than the other by say 20%. Are standards of living, or rather real wages, equal in the two regions, as a cost-of-living deflation would indicate, or are real wages higher in the higher money wage region? The Index would indicate that the latter conclusion is right, since it is the same everywhere at a given time, so that no space deflation is legitimate. In fact the worker in the high wage region is better off, since he can live in a manner identical with a worker in the lower region, retire with 20% higher savings, and live better after moving to the lower wage region (providing transport costs are not too great). Or even more clearly, he can remit part of his wage to his mother living in the lower region. If money itself is costly to transport, we must take money in the two regions to be two separate currencies, and when money wages are reduced to this single numeraire the 20% agio may prove to be illusory (see Appendix I). Further, in influencing a worker's decisions to migrate or stay at home the cost of living will of course play a role, but not as all as a deflator. The worker, if he behaves as a rational firm, will maximize his present value, and this means, approximately, maximizing the difference between his wage and his pure consumption (not productive consumption) with no interregional deflation. In the case of a firm deciding where to locate this is even clearer. (This analysis must be modified if lags and frictions exist. see Appendix III)

The quantity weights x_i in equation (5) which go into the Universal Deflator are stocks, not flows. It might be thought that this is due to our having chosen wealth to deflate, and that if we had worked with income or saving, which are flows, we would have obtained flows as weights. This is not correct. Equation (3) is the formula for the rate of wealth accumulation; it is identical in form with equation (1) for the stock of wealth, and has the identical deflator. We therefore reaffirm our conclusion that the single Universal Price Index should have stocks for weights. This is significant in view of the fact that just about every index number in use uses flows as weights--production, consumption, sales, etc.

Will the pervasive use of flow rather than stock weights introduce a bias into commonly used index numbers? It will if the ratio of flows to stocks over types of entities is correlated with price changes. The ratio flow/stock is high for perishable goods and low for durables. But there is no reason to believe that price changes are correlated with perishability. The ratio flow/stock is probably high for rapidly expanding goods. These as a group probably have declining prices because of growing economies of scale. Conversely the ratio will be low for declining goods. It is hard to say whether these as a group will have any special price movements. The second factor is the one which appears to lead to a bias, and would lead us to believe that, because of the overweighting of declining prices, most price indexes in existence have a <u>downward</u> bias.

There is another factor which is supposed to bias price indices in the opposite direction, and that is quality change. What appears to be a rise in price is actually a response to a quality improvement, and should not be counted as a price rise at all; and conversely for quality deterioration. Since the bulk of quality changes seem to be improvements, a net upward bias is introduced into the price index. The argument is valid and applies to both stock-weighted and flow-weighted indices, and hence to The Index. The remedy is a finer grading of entities: quality variants of the same type in the original grouping are to be counted as separate types.[6]

[6]as recommended by Stigler Trends in Output and Employment

What this means in practice is that entities undergoing important quality changes should be omitted from the link index. Since these would show price rises, their omission removes the upward bias from the index. In fact, however, if we had a sufficiently fine grading we would probably find the prices of goods with rapid quality improvements actually falling, since the old grade is becoming obsolete and the new grade is subject to growing economies of scale. Even complete omission, therefore, will probably still leave a residual upward bias. The correct solution, as stated above, is finer grading with complete inclusion.

The above discussion applies to what may be called tangible quality improvements. <u>Intangible</u> quality changes raise issues of a much more profound character, and will be discussed below.

EXTERNAL ECONOMIES, PRODUCTIVITY AND EFFICIENCY

We now examine the assumptions upon which the preceding analysis rests: absence of net external effects and first order homogeneity of the wealth function. We consider the entire closed world economy and count in human beings as wealth, so that these immediate objections to 1st degree homogeneity are eliminated from the start. This clears the way for the consideration of economies of scale, external economies, increasing returns, or whatever you wish to call it. These economies are to be distinguished, first of all, from external effects. There can exist external effects without increasing returns: for example, in a monopoly. Carrying through the previous analysis for the case of increasing returns leads to an inequality in place of equation (5): the change in the index in the time-direction of increasing production is less than the Divisia-Roy index, or otherwise expressed, the latter has an upward bias. This is true if no ~~whether or~~ net external effects exist. Conversely, so long as the

wealth function is 1st degree homogeneous the Divisia-Roy index number in equation (5) is the correct formula for The Index, regardless of the existence of net external effects.

This has important implications for income and wealth accounting. It means that one should proceed naively in aggregation, making no corrections for monopolistic distortions, taxes, or other interferences with competitive prices. The Universal Deflator takes all these into account and still retains year-to-year comparability.[7]

[7] cf. Musgrave Theory of Public Finance Chap9, who takes the criterion of year-to-year comparability also in evaluating different national income accounting approaches.

NOT CORRECT

So much for external effects. The problem of increasing returns, however, is critical, for on its non-existence depends the validity of the preceding analysis.

To describe the state of an economy by the quantities of the various types of entities that constitute it is to miss most of what goes under the name of organization: the spatial relations among these entities, their knowledge of each other, their coordination. These relations, which may collectively be labelled "goodwill"[8], clearly

[8] cf. Preinreich Econometrica 1940

augment income. They are sporadically capitalized by firms. They exist on a whole hierarchy of levels, from well below the plant level up through the firm, the community, the region, the nation and the world. This goodwill, however, is not some non-material entity superadded to our stocks of goods, but reflects subtle qualitative variations in these stocks themselves. Spatial relations may be accounted for by separating entities by their locations. Knowledge and skills are clearly qualitative distinctions among human beings. The upshot appears to be that a very fine grading of stocks of entities would catch all the goodwill in the world. Since these qualities are so easily overlooked and hard to pin down, we may refer to them as intangible qualities. The distinction between tangible and intangible may lie in the high degree of specificity of the latter.

If these intangible changes are ignored the comparison of outputs with inputs would superficially appear to show the existence of increasing returns, or increasing "efficiency" of resources.

Productivity and Efficiency

A productivity measure is the ratio of some output to some input. To keep numerator and denominator comparable it is advisible to take the output which is attributable to that specific input: thus, labor-output over man-hours, or property income over capital-hours. Both of these measures show secular gains. The labor productivity measure raises no difficulties of principle, being attributable to gains in the quality of labor and perhaps to the relative increase of complementary factors of production. But the capital productivity rise appears strange indeed. There are no natural units of capital as there are natural units of labor, because capital comprises a thoroughly heterogeneous group of entities. (The only

universal measure available would be tonnage, which is absurd in this context). The quantity of capital is in fact measured in real dollars. But the numerator is also measured in real dollars per unit time, and the ratio should be nothing but the rate of return to capital(if depreciation has been netted out of the numerator). The secular rise in capital productivity, if genuine, would seem to imply a secular rise in the rate of return to capital.

The rate of return to capital is not an easy thing to measure. Interest rates are not good measures because of the influence of risk, liquidity and taxation factors. The ratio of stock yields to stock prices is influenced by these factors and also by speculative instability. We can, however, estimate the rate of return to all wealth in general, counting in wealth in the form of human beings. This is simply the relative rate of growth of the economy, which has been measured at something like 3% per year on the average. (Strictly, the whole world should be brought in, but we ignore this complication). This relative rate of growth has been fairly stable secularly since 1869. This is an indication that the relative rate of return to capital has been fairly stable, though not a very strong one. There is a considerable margin of uncertainty in this growth figure, but the conclusion of stability would not be affected unless there were a secular change in the direction of the bias. But the factors which would bias the growth figure--quality improvements, accumulation of goodwill, use of flows instead of stocks--would appear to be fairly constant in their influence over time. We conclude very tentatively that there has been little or no secular rise in the rate of return to capital.

This implies that the relative rate of growth of net property income should about equal the relative rate of growth of capital, while the figures appear to show that the former exceeds the latter.

Total wealth, as estimated by the perpetual inventory method[9] is

[9]Goldsmith A Study of Saving in the United States vol I

the sum of past saving, netting out depreciation. Its rate of accumulation may be underestimated if income is underestimated and/or consumption is overestimated and/or depreciation is overestimated. As for depreciation, while it may be badly in error for any one year, its cumulation over time will be about correct, since this will be a write-off of worn out and obsolete capital. (Large changes in the price-level may introduce accounting distortions, but for secular problems we may ignore this factor). An underestimate of income would be no explanation of the secular rise in the ratio of income to capital, because it would affect numerator and denominator in the same proportion (if consumption were underestimated in the same proportion). The remaining possibility is an overestimate of consumption. The case for this is very strong. A good deal of effort goes to the creation of goodwill (in the generalized sense used above)--education, personnel work, advertising, standardization, much governmental activity. This is correctly counted in with income, but is, for the most part, also taken to be immediately consumed. That is, the intangible quality changes which are induced are ignored. These show up in the augmentation of future income, and therefore induce a secular rise in the productivity ratio.

An alternative explanation for the rise in capital productivity is

index-number bias, due, for example, to quality improvements. This fails to explain anything, however, since it would affect numerator and denominator alike in the same proportion. This might not occur if output and capital stock were estimated separately and deflated by different indices. But in fact the latteris nothing but a net sum of the former, as now calculated. We are left with the previous conclusion that the secular rise in capital productivity is to be attributed to the accumulation of goodwill not accounted for.

This specious productivity rise can be put to good use to correct for a final bias in our Index. We have seen that tangible quality improvements will lead to an upward bias, and that the remedy is a finer grading of entities. But also intangible quality changes will lead to a further upward bias. The intangible quality improvements in all entities together augment expected total income, and so lead to rises in the capitalized value of these entities. These, however, show up superficially as price rises, and thus bias The Index upward. A grading sufficiently fine to remove this bias would also account for all goodwill accumulation, and is probably not to be achieved. We can, however, measure this bias indirectly by none other than the above specious capital productivity rise: this measures the relative rate at which unaccounted-for goodwill is accumulating, hence the rate at which The Index is being incorrectly inflated.

A little algebra will clarify these complicated relations. All of the following letters represent relative rates of change, in % per year.

Suppose measured money income is growing at rate A
" " " capital " " " " B

Suppose The Index, corrected for tangible quality changes but not goodwill, is growing at rate C

The rate of growth of goodwill is A - B

The true rate of growth of The Index is then C - (A - B)

The real rate of growth of income is A - (C - (A - B)) or 2A -B - C
This is also the real rate of growth of capital.

(The above arguments slurred over the distinction between total income and property income; they stand unmodified if their ratio has been about constant over the secular period).

We conclude with a list of price index biases and their directions.

exclusion of human capital	downward ?
confinement to one conntry rather than the entire world	upward for US national indices ?
using flows rather than stocks	probably downward
quality changes	upward
goodwill accumulation	upward

Appendix I Wealth Inventories

The first problem which arises in measuring the quantities of the various types of entities is: what is an entity? Is this to be counted as one house or as a collection of five rooms? Is a railroad network a single entity? or a river? Conversely, must we count a person as a single entity?--why not as a congeries of limbs and organs, or functionally as a collection of abilities? This is the problem of how to chop up the universe into convenient pieces. Analytically, the finer things are chopped up the better, because we have all the information of the coarser breakdown and more besides. However, in the world things are bought and sold in organized lumps, so that the necessity for pricing limits our degree of refinement.

What kind of distinctions among entities should be made. Again grading should be as fine as is feasible. Distinguish things by quality variations and locational variations. The same entities which are segmented by price discrimination should be distinguished, each with its own price. Mobile transportation equipment should be evaluated where it is at the time, in accordance with its price "potential".[10] Big entities which are sold only as units (e.g. firms)

[10] Koopmans and Reiter Ch XIV Activity Analysis of Production and Allocation

have to be counted as single entities.

We select a numeraire entity arbitrarily. Change of numeraire merely changes Index and non-deflated values in the same proportions, so that real values are invariant. Arbitrage establishes a unique set of exchange rates between all entities in the world (except for the presence of lags and frictions; see Appendix III). If arbitrage is prevented by, say, political restrictions, we simply make a finer distinction of the same entity on opposite sides of the barrier.

Appendix II Regional Indices

Time-series indices are at least approximations to The Index. Cross-sectional indices do not have even this virtue. But if one insists on computing them the following considerations may be deemed relevant.

The whole set of prices and quantities of things spread out over space are in some kind of imperfect adjustment to each other. It would seem reasonable that in measuring how much higher prices are in place A than in place B one should take all prices everywhere into account. The following gives a very simple index number based on this principle.

Let x_{ij} be the quantity of goods of type i in place j
" p_{ij} " " price " " " " " " " "

The total value of good i in existence is $S_j\, x_{ij}p_{ij}$. Call this V_i.

The total quantity of good i in existence is $S_j\, x_{ij}$. Call this Q_i.

Define the average prive of good i as V_i/Q_i. Call this P_i.