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4. FEASIBILITY

A

4.1. Introduction

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We have argued that the world can be described as a measure μ over the space (Ω,Σ), where Ω is the set of all possible histories (see Section 2.5). Here, for any measurable set of histories $\underline{H}$, $\mu(\underline{H})$ is the total "mass" embodied in this set, where "mass" is to be interpreted in the broad sense discussed ~~above,~~ in chapter 2. ~~pages~~

This is the point of view of an omniscient observer who describes the world after the entire drama has unfolded itself (at time $+\infty$, so to speak). From the point of view of someone living and acting in the world, μ is not given in all detail. Rather, he has some power, or freedom, to choose how the world will develop. This may be represented formally by a set, $\mathbb{M}$, of measures over (Ω,Σ). The interpretation is that, for any $\mu \in \mathbb{M}$, there is some feasible plan of action by which he can guarantee that history will unfold according to the description μ, but that no feasible plan of action will attain any μ not belonging to $\mathbb{M}$. The set $\mathbb{M}$ will vary from person to person, and also will vary for the same person at different times. It will be called the feasible set of person $\underline{p}$ at time $\underline{t}$.

For a beggar, the feasible set will be relatively "small". That is, he has so little power that the various alternative measures μ in $\mathbb{M}$ will be "very similar" to each other; his actions make "very little" difference. For an emperor, the feasible set will be relatively "large".

There are various oversimplifications and conceptual difficulties involved in the notion of feasible set as just presented. First~~ of all~~, it neglects uncertainty. One does not actually know the full effects of any actions one might try to undertake. This uncertainty may be represented as a probability over the universe set M^o of all measures over (Ω, Σ). This is a two-level measure (see Section 2.8) representing the effect of one attempted line of action. The feasible set itself will then be a set of such two-level probability measures.

This is a fairly complicated construction, but something like it appears necessary to handle the problem of uncertainty adequately. In this chapter we shall, by and large, pass over the problem of uncertainty to avoid undue complexity. The assumption~~, then,~~ will be that one has perfect information concerning the consequences of any plan of action. The feasible set M, instead of being a set of probabilities over M^o, is then merely a subset of M^o.

~~The~~ a second difficulty concerns the interaction of several agents. Doesn't the power of person p_1 depend on the actions of other persons $p_2, p_3, \ldots$? What if they make incompatible choices?

~~Let us~~ We examine in some detail how a plan of action ~~gets~~ is translated into a measure μ, under conditions of certainty. Actions include, in the first instance, motions of the body ~~which~~ that affect the environment — planting, harvesting, and eating; weaving, carrying, and building, etc. The plan gives the time-schedule for these actions, starting from the time t_o at which the plan begins. The plan also includes actions affecting oneself, which enable one to carry out the other actions at the appropriate times, ~~— for example~~ e.g., locomotion to be at the right location for an action, self-maintenance activities, self-training regimens to develop the skills needed for some future action. These actions set up causal chains ~~which~~ that reverberate into the future. The certainty assumption means that one can predict these effects perfectly (including the unfolding of history that continues after one's death; it also means~~, by the way,~~ perfect information concerning the history of the world previous to t_o).

§12.2

Now introduce other people into the environment. Just as with the natural environment, other people are affected by one's actions, — in particular, by speech, and by the writing and sending of ~~written~~ messages. The assumption of certainty means that one can predict perfectly the responses of others to one's actions, — which may be to ignore these actions, ~~or~~ to engage in cooperative activities with oneself, ~~or~~ to attack oneself, ~~or~~ to take action affecting a third person, etc. We again have an unfolding of causal chains, except that these chains now involve the activities of other people.

Thus one's power does indeed depend on the actions of other ~~people~~ p; but since their responses are, by assumption, known with certainty, no problem of incompatible choices arises.

The assumption of certainty is ~~obviously~~ a gross simplification, yet often a useful first approximation. ~~The point is that~~ it is not necessarily a worse approximation for someone living in society than it is for a Robinson Crusoe. A high degree of predictability is a <u>sine qua non</u> for social existence, and one of the prime functions of social institutions is to insure this predictability. We know that, under normal conditions, a storekeeper will sell us any item on the shelf at its stated price, a fire department will respond to an alarm, an oncoming motorist will yield our right of way. An employer knows his employee will follow orders within a certain broad zone of "legitimate" authority. In fact, the general course of civilization has probably been to increase the overall predictability of the future. The incursion of droughts, floods and other natural fluctuations has been damped; epidemics are less threatening. The improvement of transportation and the rise of insurance – both private and social – pools the risks of individual misfortune over the entire society, and establishes a subsistence floor ~~which~~ that tends to rise over time. Violence tends to decline <u>within</u> a region with the territorial spread of the nation-state: Philadelphia and New York do not make war on each other as Athens and Sparta did.[1]

Assuming certainty, then, our problem in this chapter will be to describe feasible sets **M** in various plausible and convenient ways. $\mathcal{M}$ is a very complicated object even under certainty (remember that each of its points is a complete description of the world), and one needs good schematic ways for characterizing it, at least approximately.

In general, we ~~shall~~ try to characterize $\mathcal{M}$ by "whittling down" from above; i.e. ~~That is~~, there will be a number of simpler sets, each one consisting of all measures satisfying some test criterion. Any feasible measure must satisfy all ~~of~~ these tests, so that $\mathcal{M}$ is included in the intersection of all these sets. Hopefully, it is equal to their intersection; if not, further tests are needed.

Thus, a given measure μ may be infeasible because it violates a natural law; ~~or,~~ because of technical ignorance; ~~or,~~ because resources are not available; ~~or,~~ because it violates a legal statute; or, because the person lacks the money or authority to induce needed actions by other people. Then $\mathcal{M}$ will ~~then~~ consist of the measures μ satisfying all these criteria (and perhaps other ~~criteria~~ not listed).

Several of these broad criteria could themselves be expressed as the conjunction of simpler ~~criteria~~ ones. Thus to satisfy the natural-law criteria, the measure may have to satisfy conservation laws, maximal density constraints, dynamical laws in the form of differential equation systems, etc. To comply with legal statutes, it must satisfy zoning laws, traffic laws, housing laws, anti-pollution laws, etc.

Our aim~~, then,~~ is to make a quick survey of these various exclusion criteria. Obviously, this ~~survey~~ must be superficial; anything more would require expertise in dozens of different specialties. ~~Rather,~~ We stress features ~~which~~ that are analytically tractable, and at the same time catch broad structural aspects of the various criteria. In particular, these will include most of the feasible sets used in the rest of ~~this~~ the book. (Budget constraints will be discussed ~~later,~~ in chapter 6).

(A)

4.2. Uncontrollable Regions

Person p at time t_o has feasible set $\mathbb{M}$. He cannot do anything about the world before time t_o; The past has already happened. How is this fact reflected in the set $\mathbb{M}$?

Let μ' and μ'' ~~both~~ be feasible. Recall that the cross-sectional measure at time t gives the distribution of mass over $R \times S$ at that time. It must be true that, for any $t < t_o$, the cross-sectional measures at t determined by μ' and μ'' are the same. Thus

$$\mu'\{h|h(t) \in E\} = \mu''\{h|h(t) \in E\} \qquad (4.2.1)\ (1)$$

for any measurable $E \subseteq R \times S$, any $t < t_o$, and any two $\mu', \mu'' \in \mathbb{M}$. A similar equality holds for double-cross-sectional measures, etc., provided all times are in the past of t_o. Next, consider the production and consumption measures, which are on universe set $R \times S \times T$ (~~pages ______~~ Section 2.5). If we consider the past half-space

$R \times S \times \{t | t < t_o\}$, then the production measures λ_1' and λ_1'', derived from μ', $\mu'' \in \mathcal{M}$, respectively, must be identical when restricted to this half-space, since they both equal the actual pattern of production realized in the past. Similarly for consumption measures λ_2', λ_2''.

All these remarks are implied by the following principle, which expresses with complete generality the notion of the uncontrollability of the past. It is convenient here to add the artificial point z_o to $R \times S$, and let the history h take on the value z_o at the times before it is "born" and after it "dies." With this convention h has as its domain the entire time axis T, with range in $(R \times S) \cup \{z_o\}$.

Two histories, h' and h'' are <u>identical before t_o</u> iff $h'(t) = h''(t)$ for all $t < t_o$. Let H be a set of histories. H is <u>t_o-past-determined</u> iff, whenever $h' \in H$ and h' and h'' are identical before t_o, then $h'' \in H$. We now state the Past Uncontrollability Principle:

Any two feasible measures have identical values on all measurable t_o-past-determined sets.

The set appearing in (1) is t_o-past-determined. So is the set determining production or consumption on any measurable $G \subseteq [R \times S \times \{t | t < t_o\}]$. This shows that the remarks above are implied by the Past Uncontrollability Principle.

Not only the past, but a portion of the future, will be uncontrollable. Consider a fire station located at s_o at time t_o. The locations ~~which~~ That can be reached by time $t > t_o$ by a fire engine starting at t_o depend on the maximal speed of the engine, street layout, traffic congestion, etc. There will be an accessible region ~~which~~ that in general expands with increasing t, the whole set of accessible points in Space-Time forming roughly a "cone" with its apex at (s_o, t_o), and opening into the future.[2]

Without speedier communications, the entire subset of $S \times T$ outside this cone is uncontrollable by the engine dispatcher at time t_o. Maximal speed limitations imply that such uncontrollable regions exist in general.

Even within this cone there will be aspects ~~which~~ that are virtually uncontrollable. The great processes of nature, earthquakes, hurricanes, etc., are still in this category. For all but a handful of people, the tides in the affairs of men — war, revolution, depression, religious movements, fashion cycles, ~~etc.~~ — must be considered uncontrollable.

We conclude with an abstract definition of uncontrollability ~~which~~ that includes as special cases everything discussed in this section.

Definition: Let $\mathcal{M}$ be a set of measures on the space (A, Σ). Set $E \in \Sigma$ is uncontrollable with respect to $\mathcal{M}$ iff $\mu(E)$ has the same value for all $\mu \in \mathcal{M}$.

4.3. Cross-Sectional Constraints

Cross-sectional constraints are those ~~which~~ that exclude measures whose cross-section as of time t fails to satisfy certain conditions. In more detail, we start with a measure μ on the space of histories (Ω,Σ), and then examine its cross-section μ_t, which is given by

element

$$\mu_t(E) = \mu\{h|h(t) \in E\}$$

for all measurable E. μ_t is, of course, a measure on the space $(R \times S, \Sigma_r \times \Sigma_s)$. There will in general be many measures μ with the same cross-section at t, ~~and~~ if μ_t fails, ~~then~~ all ~~of~~ these measures are infeasible. Some of the conditions to be imposed may have to be met by μ_t for all time-instants t, others perhaps for only some t.

To simplify notation we ~~shall~~ drop the subscript t. The measures are still over universe set $R \times S$.

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B

Integer Values and Finite Concentration

For a certain subset, R', of Resources, in which objects come in "natural units" — such as people, cattle, and cars — μ may take on only integer values (or be infinite) when restricted to $R' \times S$. The constraint here is one of purely semantic origin; Any other measures would be meaningless.

D

Definition: Measure μ on (A,Σ) is concentrated on set $E \subseteq A$ iff $\mu(A) > 0$ and $\mu(F) = 0$ for all measurable F disjoint from E (E itself need not be measurable).

We have already met several related concepts. Measure μ is simply-concentrated iff it is concentrated on some singleton set. Similarly, μ is finitely concentrated iff it is concentrated on some finite set.

Restrictions to finitely concentrated measures arise in two ways. First, certain resource-types are just naturally not "spread-out" over space, and are well represented as being confined to a finite number of locations. One thinks of the natural units mentioned above, and perhaps others.[3] Second, even though it is technically feasible to spread a resource continuously over Space, ~~one's~~ budget may not permit this; e.g. ~~For example,~~ there may be an overhead cost associated with each location at which the resource is situated. In many problems the multiplicity is also specified; e.g. ~~for example~~, given n policemen, deploy them ~~so as~~ to minimize total crimes. The case $n = 1$ is especially interesting; the classic plant-location problem of Weber is a special case.

There is a close connection between integer-valuedness, atomicity, and finite concentration. Recall that μ is atomic iff $\mu(A) \neq 0$, and, for all $E \in \Sigma$, either $\mu(E) = 0$ or $\mu(A \backslash E) = 0$. Every simply-concentrated measure is atomic.

Theorem: Let μ be a bounded measure, not identically zero, on (A,Σ). If μ takes on just a finite number of values, then there is a finite measurable partition $\{A_1,\ldots,A_n\}$ such that μ restricted to each A_i is atomic.

Proof: Since μ is bounded, ~~there is~~ a countable measurable partition $\{A_0, A_1, \ldots\}$ exists such that μ restricted to A_0 is non-atomic, and μ restricted to A_i is atomic for $i = 1, 2, \ldots$. This partition must in fact be finite, since otherwise $\mu(A_1)$, $\mu(A_1 \cup A_2), \ldots$ would give an infinite number of different μ-values. On the non-atomic part, μ takes on all values between 0 and $\mu(A_0)$. Hence $\mu(A_0) = 0$. Since $\mu \neq 0$, there is at least one A_i, say $i = 1$, and μ restricted to $A_0 \cup A_1$ remains atomic. $\square$

A_0 zero

The converse of this theorem is also true, but of less interest to us. Now if μ is integer-valued and bounded (we ~~shall~~ do not consider the unbounded case), it takes on just a finite number of values, hence is atomic on all A_i for some partition $\{A_1, \ldots, A_n\}$. Atomic measures are not always simply-concentrated; however, there is one very common condition under which the two concepts coincide.

Definition: Let (A, Σ) be a measurable space. Σ is <u>countably generated</u> iff there is a countable subclass $\mathcal{G} \subseteq \Sigma$ ~~which~~ that generates Σ.

For example, the Borel field on the real line is countably generated, since it is generated by the collection $\{a \mid a < x\}$, x rational. Similarly for the n-dimensional Borel field, $n = 2, 3, \ldots$. But this property will often not hold for more complex ~~sigma~~ σ-fields, such as the one over the space of histories Ω, or those involved in multi-layer measures.

Theorem: If μ is an atomic measure on (A, Σ), and Σ is countably generated, then μ is simply-concentrated.

Proof: Let $\{G_1, G_2, \ldots\}$ generate Σ. For each G_n, exactly one of $\mu(G_n)$, $\mu(A \setminus G_n)$ is positive, the other being zero. Let F_n be either G_n or $A \setminus G_n$, chosen so that $\mu(F_n) > 0$, $\mu(A \setminus F_n) = 0$. Let $\cap_{n=1}^{\infty} F_n$. $\mu(A \setminus F) = \mu[\cup_{n=1}^{\infty}(A \setminus F_n)] \leq \mu(A \setminus F_1) + \mu(A \setminus F_2) + \ldots = 0$, so that $\mu(A \setminus F) = 0$. Hence $\mu(F) > 0$.

F is therefore non-empty, so there exists $a_o \in F$. We show that μ is concentrated on the set $\{a_o\}$. It suffices to show that, for any measurable E with $a_o \notin E$ we have E, F disjoint; for then $\mu(E) = 0$ is immediate. Consider, then, the class Σ' of all measurable sets which either contain F or are disjoint from F. Σ' is closed under complements and countable unions, so that it is a sigma-field. Furthermore, $G_n \in \Sigma'$ for all n. It follows that $\Sigma' = \Sigma$. Since E above does not contain F, it is disjoint from F, and the proof is complete. |||

We conclude that, if μ on (A, Σ) is integer-valued and bounded, and Σ is countably generated, then μ is finitely concentrated. Thus, in most cases of interest, integer-valuedness is a strengthening of the finite concentration condition.

B

Space Capacity

Let $r \in R$ be some particular resource-type and let F be a region of Space. The amount of r that can be squeezed into F may well have a finite upper limit, where we ~~One~~ "run~~s~~ out of space." ~~at this limit.~~ Let $\nu_r(F)$ be this upper limit.

If F_1 and F_2 are two disjoint regions, we must have

$$\nu_r(F_1 \cup F_2) \leq \nu_r(F_1) + \nu_r(F_2). \qquad (4.3.1)\ ~~(1)~~$$

For if (1) were false, there would be no way to approach capacity on $F_1 \cup F_2$ without exceeding capacity on one of the two subregions. Condition (1) is called finite subadditivity. It is possible that the inequality in (1) may sometimes be strict, which means that capacity cannot be reached in F_1 and F_2 simultaneously. The most interesting case, however, is where (1) becomes an equality for all disjoint regions F_1, F_2, so that ν_r is finitely additive. ~~In fact,~~ it is not unreasonable that ν_r should be countably additive, so that for any countable packing of regions, the capacity of the union is the sum of the capacities of the individual regions. We may also safely assume that $\nu_r(\emptyset) = 0$. With these assumptions, ν becomes a measure over Space, the capacity measure for resource r.

Now let μ be a cross-sectional measure. We assume that all singleton sets are measurable in R: $\{r_o\} \in \Sigma_r$, for all $r_o \in R$. It must then be true that

$$\mu(\{r\} \times F) \leq \nu_r(F) \qquad (4.3.2)\ ~~(2)~~$$

no qu on

for all resources r and regions F. Condition (2) states that the total mass of r in region F does not exceed the capacity of that region for r. But (2) is not ~~stringest~~ stringent enough. Each resource-type is taking up space on its own, and the region must have a global capacity sufficient to accommodate them all simultaneously. This suggests the following approach.

First, we postulate a general capacity measure α over Space, (S,Σ_s). This measure will be called **ideal area**. It may or may not coincide with ordinary physical area (or volume). Next we postulate a function, $f: R \times S \to$ reals, having the interpretation that $f(r,s)$ is the "amount of space" needed per unit of resource r at location s. f is non-negative and assumed to be measurable with respect to $\Sigma_r \times \Sigma_s$. This description of f is vague; the precise rôle of f (and α) is given in the next inequality, which gives the total system of capacity constraints ~~which~~ that must be satisfied by any feasible cross-section μ.

$\int_{R\times F} f\,d\mu$

$$\int_{R\times F} f\,d\mu \leq \alpha(F), \qquad (4.3.3)\ \cancel{(3)}$$

for all regions F. The left~~-hand~~ side of (3) is a plausible expression for the total "demand for space" by all resources together ~~which~~ that occupy region F, and (3) requires that μ be small enough so that this total demand does not exceed the "space" available in any region.

The most interesting special case arises when f does not ~~in fact~~ depend on its s-coordinate: $f(r,s) = f(r,s')$ ($=f(r)$, say) for all $r \in R$, $s, s' \in S$. $f(r)$ may then be thought of as the reciprocal of the maximal density to which resource r can be squeezed. If, in addition, just one single resource-type r_o is distributed over Space (~~that is~~ i.e., $\mu[(R\setminus\{r_o\}) \times S] = 0$), then (3) reduces to (2):

$$\int_{R\times F} f\,d\mu = \int_{\{r_o\}\times F} f\,d\mu = f(r_o)\mu\left(\{r_o\} \times F\right) \le \alpha(F),$$

which is the same as (2) if we define $\nu_r = \alpha/f(r)$.

Note that (3) allows the possibility of f being zero sometimes. A resource-type r for which $f(r,s) = 0$, all $s \in S$, will be called <u>non-space-using</u>. Postulating that certain resources are non-space-using is a simplifying approximation ~~which~~ That is often useful. In this case the constraints (2) simply disappear.

~~Let us~~ consider some real-world examples of maximal capacity constraints. ~~First,~~ for most resource-types there will be some physical density beyond which the resource will be destroyed. Here α will be ordinary physical volume (or perhaps area). For people this capacity is sometimes approached: (3) must be near equality inside subway trains during the New York City rush hours.[4]

Legal statutes often have the effect of placing capacities well below the physical limit. These may be referred to as <u>anti-congestion laws</u>, whether that is their primary purpose or not. Fire laws will limit occupancy of halls; no-standing laws will limit occupancy of buses and movies. Zoning and housing laws requiring open spaces, minimal lot sizes, maximal lot coverage, maximal building height and bulk, etc., all have the effect of spreading people out and reducing actual occupancy far below what would be physically possible.

The fact that we have here social rather than physical constraints raises no difficulties. The interpretation of (3) is different: The measures μ ~~which~~ that violate it are illegal, not necessarily physically impossible. The function f used in these constraints will in general be larger than for the physical constraints; more space is legally required per unit resource than is physically necessary. ~~Also,~~ Since laws vary from place to place, $f(r,s)$ will in general vary with its s-coordinate (rising in places where laws are ~~"together"~~ "tougher"). For physical constraints one can probably make do with an f depending only on r.

§12-6 cont'd

There is one essential difference between natural laws and statute laws as far as feasibility is concerned. Natural laws cannot be violated (by definition), but one can often violate statutes (by committing a crime). Thus to treat statutes as constraints is to restrict the feasible set unduly; e.g., one may decide to park illegally and run the risk of getting a ticket~~, for example~~. For the most part we ~~shall~~ ignore this point and treat statutory "constraints" as binding.

Exclusions are a limiting case of maximal capacity constraints. They specify that a measure must take the value zero on certain sets. Examples are laws against trespassing, zoning laws, segregation laws, and curfews. ~~Take~~ Consider a curfew, for instance. It specifies that $\mu(E \times F) = 0$, where E is the set of "unauthorized personnel" and F is, say, the streets of a certain town. Cross-sectional measures μ_t must satisfy this condition for those times t at which the curfew is in effect — say night-times over a certain time-interval.

Exclusions may be handled formally by letting f take on the value $+\infty$ in (3). If α is sigma-finite (as we may assume from its interpretation), it is easy to show that (3) implies $\mu\{(r,s)\,|\,f(r,s) = \infty\} = 0$. Thus all that needs doing is to set f equal to infinity on the excluded sets.

Finally, the limited variety constraint is a special kind of exclusion. Some resource-types may not be able to exist. Commodities may not be producible except in a limited number of qualities and styles; unicorns and philosophers' stones are not found in nature, etc. Let E (assumed measurable) be the subset of R consisting of all these excluded resources. Then $\mu(E \times S)$ must be zero for any feasible cross-section μ.

Resource Capacity

The quantities of various resource-types available may be limited. These limitations may change over time, as resources are created or destroyed, but for any given time t we may postulate a resource-capacity measure ν_t on the space (R, Σ_r). Any feasible cross-section μ_t must then satisfy

$$\mu_t(E \times S) \leq \nu_t(E) \qquad (4)$$

for all $E \in \Sigma_r$, i.e. ~~That is~~, the left marginal of μ_t cannot exceed ν_t.

Limited variety constraints are a special case of (4) (as well as a special case of exclusions). They may be represented by setting ν_t equal to zero on the appropriate sets.

There will in general also be constraints of the form (4) for sub-regions, and not merely for Space as a whole. This will occur whenever resources are tied up in particular regions and cannot be transported elsewhere with infinite speed.

(B)

Disallowed Configurations

Recall that a configuration is simply a measure on universe set $R \times F$, R being the set of resources and F a region of Space. We have defined when two configurations are to be considered of the same type, and also have defined the notion of an abstract configuration-type, which may be exemplified in various actual regions (See Section 2.7). (These concepts involve a metric on Space).

~~Now~~ suppose that certain abstract configuration-types are set aside as "disallowed". These ~~are configurations which~~ that would, if exemplified, violate some natural or human law. For example, the following might be illegal configurations: two bars within distance x_1, ~~or~~ a bar and a church within distance x_2, or a house without a fire hydrant within distance x_3.

A cross-sectional measure μ is to be considered infeasible if, for any region F, μ restricted to $R \times F$ exemplifies a disallowed configuration-type. Thus a separate test must be passed for each region.

A less stringest version of this test works with a set of "allowable" abstract configuration-types. A cross-section μ passes the test if there is a countable collection of regions, $\mathcal{F}$, which together cover S, such that μ restricted to $R \times F$ exemplifies an allowable configuration type for each $F \in \mathcal{F}$. That is, it must be possible to represent μ as a "patching" of allowable configurations.

These two versions will be called the strict and loose constructionist versions of the configuration test, respectively.

This approach, in either version, is very general and flexible. However ~~On the other hand,~~ it does not allow for spatial variation in what is allowable or is not; thus it is probably most useful in connection with natural laws, or within the domain of a single legal system.

4.4. Intertemporal constraints

We now go on to feasibility conditions involving several different time-instants. The possible feasibility conditions are much richer than for cross-sections. One broad class of conditions requires that certain sets of histories have measure zero. This resembles the concept of exclusion ~~which we have~~ discussed above. In fact, exclusions are just a special case, requiring that the set of histories occupying $G \subseteq R \times S$ at time t have measure zero.

Non-Interactive Systems

~~Let us~~ We start with a very simple case. Suppose for each pair of positive numbers t_0, t_1 ($t_0 < t_1$), there is a function $f_{t_0 t_1} : R \times S \to R \times S$, expressing a dynamical law: A history ~~which is~~ in state (r,s) at moment t_0 will move to state $f_{t_0 t_1}(r,s)$ at moment t_1.[5] This means that the entire future of a history is determined by the state it occupies at any one moment. ~~One~~ We then specif~~ies~~y that the set of all histories violating this dynamical law has measure zero (assuming this set is measurable).

A condition of this sort is implausible because it does not allow for interaction: the course of a history depends only on its past, not on the environment. More generally, ~~one~~ we look~~s~~ for a rule by which the "rate of change" of a history in state (r,s) at time t depends also on the cross-sectional measure at time t. (This would require putting some structure on R ~~so as~~ to define the notion of a rate). For example, the acceleration of a particle under gravity depends on the distribution of mass over Space; the behavior of a person may depend on his observation of the distribution of behavior of other people.

Barriers

Of the innumerable forms such a rule might take, we mention just one type. A barrier for a certain state (r,s) is a configuration ~~which~~ prevent~~s~~ing that state from changing in certain ways. Barriers are quite pervasive; some occur in nature, some are added by man, some are removed by man. A house serves, ~~among other things,~~ as a barrier against the weather, preventing air in an unpleasant state from gaining access. Umbrellas are barriers against rain, thimbles against needles. The skin is a barrier against infection. Walls, fences, locks, guards and watchdogs are barriers against trespassing.

Transportation construction in general may be thought of as barrier removal or barrier circumvention. Barrier removal occurs when the rough surface of the earth is smoothed, as in road and rail construction, tunneling and bridging. ~~and dridging.~~ Barrier circumvention occurs when an alternative medium is developed enabling one to bypass the former barrier — as in air and sea travel, pipelines and powerlines, broadcasting.

By slight extension of the meaning, one can speak of barriers to entering certain occupations, certain industries, or certain social statuses (citizenship, marriage, political office, etc.) The formal analyses of these situations is similar to that of barriers in the strict sense.

B

A Pollution Model

The following model reverts back to the assumption of no interaction with the environment, but allows several histories to spread out from single points. Models of this sort may be suitable as representations of the diffusion and transformation of substances, as in air pollution studies. Our aim, however, is mainly to illustrate how the concepts we have been working with might be applied, and we make no attempt to take realistic complications into account.[6]

We start with our basic sets of Resources, Space, and Time — (R, S, T) — with ~~sigma~~ σ-fields Σ_r, Σ_s, Σ_t, respectively. Suppose a unit mass of resource r_0 is released at location s_0 at instant t_0. What has happened to it by time t_1? We assume ~~that~~ the answer is given by a function $f: R_0 \times S_0 \times T_0 \times T_1 \times (\Sigma_{r_1} \times \Sigma_{s_1}) \to$ reals.

Zero where inked

~~Let us pause for a moment~~ to explain notation. ~~First,~~ subscripts 0 and 1 will be used with R, S, and T, and points belonging to them, to distinguish "origins" from "destinations". Origins, denoted by 0, are points in $R \times S \times T$ at which mass is released into circulation; destinations, denoted by 1, are points where mass is found after circulating awhile. Also, $\Sigma_{r_1} \times \Sigma_{s_1}$ is parenthesized because it is a product ~~sigma~~ σ-field, while the other crosses stand for cartesian products.

The function f is a conditional measure; i.e. ~~that is,~~

(i) for fixed r_0, s_0, t_0, t_1, $f(r_0, s_0, t_0, t_1, \cdot)$ is a measure on the space $(R_1 \times S_1, \Sigma_{r_1} \times \Sigma_{s_1})$; ~~and~~

(ii) for fixed $G \in \Sigma_{r_1} \times \Sigma_{s_1}$, $f(\cdot,\cdot,\cdot,\cdot, G)$ is a measurable function with respect to its domain space, $(R_0 \times S_0 \times T_0 \times T_1, \Sigma_{r_0} \times \Sigma_{s_0} \times \Sigma_{t_0} \times \Sigma_{t_1})$.

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The interpretation of f is this:

$f(r_0, s_0, t_0, t_1, G)$ is the total mass in resource-location states in G at moment t_1 which arises from the release of unit zero where marked

mass of resource $\underline{r}_\theta$ at location $\underline{s}_\theta$ at moment $\underline{t}_\theta$. That is, the unit mass will become diffused, perhaps spreading over various locations, and also perhaps becoming changed into different resource-types. Thus we get a changing series of cross-sectional measures, depending on $\underline{t}_1$, the moment of observation.

¶ 12.9

We require that $\underline{f}$ satisfy the following consistency condition: If $\underline{t}_\theta < \underline{t}_1 < \underline{t}_2$ are three moments, then

on all on one line

$$\underline{f}(\underline{r}_\theta, \underline{s}_\theta, \underline{t}_\theta, \underline{t}_2, \underline{G}) = \int_{\underline{R}_1 \times \underline{S}_1} \underline{f}(\underline{r}_1, \underline{s}_1, \underline{t}_1, \underline{t}_2, \underline{G})\, \underline{f}(\underline{r}_\theta, \underline{s}_\theta, \underline{t}_\theta, \underline{t}_1, \underline{dr}_1, \underline{ds}_1) \qquad \text{(4.4.1)} \; \cancel{(1)}$$

for all $\underline{r}_\theta \in \underline{R}_\theta$, $\underline{s}_\theta \in \underline{S}_\theta$, $\underline{G} \in \Sigma_{\underline{r}_2} \times \Sigma_{\underline{s}_2}$. The left-~~hand~~ side of (1) gives the mass on set $\underline{G}$ at time $\underline{t}_2$. The right-~~hand~~ side gives the same thing indirectly: first, by finding the entire distribution on $\underline{R}_1 \times \underline{S}_1$ (at the intermediate time $\underline{t}_1$); finding the contribution to $\underline{G}$ at $\underline{t}_2$ by unit mass at $(\underline{r}_1, \underline{s}_1)$ at $\underline{t}_1$; and then integrating (in effect, taking the limit of the weighted sum of these contributions).[7]

set

If $\underline{t}_1 < \underline{t}_\theta$, then $\underline{f}$ is identically zero: there is no mass before the date of release. If $\underline{t}_1 = \underline{t}_\theta$, then $\underline{f}$ is simply concentrated, ~~located~~, with unit mass concentrated at the single point $(\underline{r}_\theta, \underline{s}_\theta)$: This merely gives the initial condition at $\underline{t}_\theta$.

The conservation of matter is expressed by the condition:

$$\underline{f}(\underline{r}_\theta, \underline{s}_\theta, \underline{t}_\theta, \underline{t}_1, \underline{R}_1 \times \underline{S}_1) = 1 \qquad \text{(4.4.2)} \; \cancel{(2)}$$

zero

for all r_0, s_0, t_0, t_1 for which $t_1 \geq t_0$. (Condition (2) states that the same total mass is present in some form somewhere at any time-instant after release. ~~On the other hand, one~~ However we may want to incorporate the gradual absorption, or "death", of some of the histories starting at (r_0, s_0) at time t_0. In this case the value of f in (2) will be a non-increasing function of t_1, holding r_0, s_0, t_0 fixed, and equalling one at $t_1 = t_0$.[8]

8

The second half of this model is a measure ν on the space $(R_0 \times S_0 \times T_0, \Sigma_{r_0} \times \Sigma_{s_0} \times \Sigma_{t_0})$, representing the pattern of release of resources (pollutants). On rectangles the interpretation of ν is as follows: $\nu(E \times F \times G)$ is the total mass of resources of types E released in region F in period G. Note that ν can incorporate the possibility of positive quantities emanating at single locations (Con Edison plants?) as well as continuous releases over Space, and also the possibility of positive "gobs" appearing at single time-instants, as well as continuous releases over Time.

X Given the measurable set $G \subseteq R_1 \times S_1$, what is the total mass embodied in the resource-location pairs in G, at moment t_1, as a result of the release pattern ν? The answer is

129 ~~66~~ 66 184

$$\lambda(t_1, G) = \int_{R_0 \times S_0 \times T_0} f(r_0, s_0, t_0, t_1, G)\, \nu(dr_0, ds_0, dt_0). \qquad \text{(4.4.3)} \ \ (3)$$

Here again (3) may be thought of as the limit of a weighted sum of the contributions to G from the various triples (r_0, s_0, t_0), the weights being determined by ν. For fixed t_1, raised dot

$\lambda(t_1,\cdot)$ is in fact a measure on $R_1 \times S_1$. If the conservation law (2) is in effect, it follows from (3) that

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$$\lambda(t_1,\, R_1 \times S_1) = \nu\Big(R_\theta \times S_\theta \times \{t \mid t \leq t_1\}\Big).$$

zero

That is, the total mass found at moment t_1 equals the total mass released not later than t_1, as one would expect. One can go a step further and define a total "exposure" measure ρ over $R_1 \times S_1 \times T_1$:

146 27 204 12 pt brackets

$$\rho(B) = \int_{T_1} \lambda\Big(t_1,\, \{(r_1,s_1) \mid (r_1,s_1,t_1) \in B\}\Big)dt_1, \qquad (4) \quad (4.4.4)$$

19

for all $B \in \Sigma_{r_1} \times \Sigma_{s_1} \times \Sigma_{t_1}$.[9] The integration in (4) is with respect to ordinary Lebesgue measure on the real line t_1, representing "quantity of time". The interpretation of ρ on rectangles is as follows: $\rho(E \times F \times G)$ equals total mass-time of exposure to resources (*i.e.*, pollutants) of types E in region F during period G. ~~Note that,~~ Unlike most of our other measures, ρ is in mass-time units (e.g., ton-hours, man-days) rather than in mass units. ρ is needed to evaluate cumulative exposure effects.

Let us give a plausible concrete example for the "diffusion" function f. We simplify by assuming that just a single resource-type is involved, and ~~that~~ no transformations in R occur, i.e., ~~That is,~~ no chemical transformations occur, but only spatial motions. This simplification is reflected formally by dropping all references to R in the preceding expressions: f becomes a function with domain $S_\theta \times T_\theta \times T_1 \times \Sigma_{s_1}$, ν becomes a measure on $S_\theta \times T_\theta$, etc.

zero

9.12.10

Space is taken to be the plane, with cartesian coordinates (x,y). Σ_s is the Borel field for the plane. With these preliminaries, we now give f in the form of an indefinite integral:

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$$f(x_\theta, y_\theta, t_\theta, t_1, G) = \iint_G \frac{1}{2\pi ab(t_1-t_\theta)} \exp\left[-\frac{(x_1-x_\theta-c(t_1-t_\theta))^2}{2a^2(t_1-t_\theta)} - \frac{(y_1-y_\theta)^2}{2b^2(t_1-t_\theta)} - k(t_1-t_\theta)\right] dx_1\, dy_1 \qquad \text{(4.4.5)} \; \cancel{(5)}$$

for all $(x_\theta, y_\theta) \in S_\theta$, all $t_\theta < t_1$, all $G \in \Sigma_{s_1}$. The integration in (5) is with respect to ~~two~~ 2-dimensional Lebesgue measure. Here "$\exp[z]$" stands for e^z; a, b, c, k are real constants satisfying: $a > 0$, $b > 0$, $k \geq 0$. Formula (5) is valid only for $t_1 > t_\theta$. We have already mentioned that for $t_1 < t_\theta$, f is identically zero; and for $t_1 = t_\theta$, f is simply concentrated with unit mass concentrated at (x_θ, y_θ). For any $t_1 > t_\theta$, then, (5) is a bivariate normal distribution with mean at $[x_\theta + c(t_1-t_\theta), y_\theta]$, variance of $a^2(t_1-t_\theta)$ in the x-direction, variance $b^2(t_1-t_\theta)$ in the y-direction, and covariance zero. The mass over all ~~of~~ Space at moment t_1 is equal to ~~M~~ $\exp[-k(t_1-t_\theta)]$, so that the conservation law (2) is satisfied iff $k = 0$.

~~One~~ We might interpret (5) as follows. A unit mass released at location (x_0, y_0) at time t_0 is subjected to random and systematic forces. The latter consists of wind blowing in the x-direction at velocity c. The former causes the mass to spread out in a normal distribution pattern. The whole distribution moves with the wind and also keeps spreading, variances being proportional to elapsed time. The k term is thrown in to allow for possible disappearance of mass through absorption.

zero

It may be verified that f defined by (5) satisfies the Chapman-Kolmogorov equation (1) (with R being deleted) ~~from this equation, of course~~.

The model we have ~~just~~ outlined is one fragment of a larger system. The release measure ν, ~~for example~~ e.g., will in its turn be derived from the distribution of activities over S × T, ~~together~~ along with their associated production measures. Conversely, the resulting measures λ given by (3) condition the environment, and thereby affect the feasibility of activities that might run in various places. This whole system of relations provides a test that must be passed by any feasible measure μ on the space of histories.

A

4.5. Activity Distributions

The feasibility test to be presented ~~in this section~~ now constitutes a generalization of the standard model of activity analysis.[10] We ~~shall~~ here concentrate on the formal development.

10

We have already discussed a "configuration" test for cross-sections μ_t, in which either μ_t restricted to any region must not be disallowed (strict constructionism), or μ_t must be "patched" together by allowable restrictions (loose constructionism). The same sort of test could be constructed for intertemporal feasibility, with activity-types taking the place of configuration types.

Tests of this sort, however, are rather clumsy to work with because ~~one~~ we must compare patterns spread over Space (or Space-Time). It would be much more convenient if the test involved only the comparison of single points, so to speak. The following is an attempt to carry out this construction.

We postulate a set of allowable activity types, Ω. In accordance with our aim, we consider only "simply-located" activity-types ~~— that is,~~ i.e., activities ~~which~~ have just a single location at any one moment. We ~~shall~~ also restrict our attention to sedentary activities, so that the single location is fixed over Time. This second assumption is less crucial and could be relaxed, but it is convenient.

Q boldface

Let us spell out what these restrictions amount to. Taking an activity to be a measure over a set of histories, these histories will all have an identical, constant, itinerary. Hence the histories are distinguished only by their transmutation paths. We may then simply identify an activity as a measure over the universe set Ω_r of all transmutation-paths (= functions whose domain is a closed time-interval, and ~~which~~ that take values in R).

9.12.11

Another concept of activity identifies it with a production or consumption measure (or the pair of them). These are over a subset of R × S × T. But because of the very special kinds of activities we are considering, all reference to S may be suppressed. For simplicity we ~~shall~~ always take the universe set to be R × T.

The distinction between activity and activity-type has been slurred over in the preceding discussion. Recall that an activity is defined in terms of "real" Space and Time, while an activity-type is defined with "abstract" sets, S' and T', in place of these (S' is a metric space and T' has the structure of the real line). An activity-type is then exemplified in an actual activity iff there is a measure-preserving spatial isometry and time translation between the "abstract" and "real" spaces.

In the present context the situation is much simpler. Since the regions in which activities are located are single points, there is no isometry problem and we can ignore metrical considerations entirely. We could still use an abstract Time set, but ~~actually~~ it will be more convenient not to do so. Finally, no confusion will arise in ~~the~~ this present discussion if we drop the distinction between "activity" and "activity-type", and simply refer to them both as "activity".

We summarize this discussion in the following definition, which combines the various special activity concepts.

Definition: An activity is a triple, consisting of a measure ρ on the space of transmutation-paths, (Ω_r, Σ'), and a pair of measures, λ_1 and λ_2, on $(R \times T, \Sigma_r \times \Sigma_t)$.

Here ρ gives the amounts of "capital goods" and "materials" tied up in the activity, while λ_1 and λ_2 are the production and consumption measures, respectively. We need not be concerned with the nature of the ~~sigma~~ σ-field Σ' on Ω_r; for the present discussion, it suffices to know that it exists. It is understood that this definition is only for the present discussion; in other cases ~~one~~ we may wish to revert to the more general activity concept discussed in chapter 2.

We ~~shall~~ use the letter q to designate an activity, and write, ~~for example~~ e.g., $\lambda_2(q,G)$ for the value of consumption in activity q on set $G \in \Sigma_r \times \Sigma_t$. As stated above, Q will designate the set of allowable activities (which in general will be a small subset of the set of all triples of measures $(\rho, \lambda_1, \lambda_2)$).

Q itself will now be made into a measurable space by placing a ~~sigma~~ σ-field Σ_q on it. The conditions to be placed on Σ_q will be indicated below.

Definition: An assignment is a measure ν on the space $(S \times Q, \Sigma_s \times \Sigma_q)$.

Assignment ν describes how activities are distributed over the world. On rectangles it may be interpreted as follows:

$\nu(F \times G)$ is the total amount of activities of types G situated in region F. This somewhat vague characterization will be pinned down below. Note that ν is a (generalized) two-layer measure, since the elements of Q are themselves (triples of) measures.

We now give the basic mathematical justification for the procedures to be used in this section. We state it in abstract form to make it self-contained.

Theorem: Let (Q, Σ_q), (S, Σ_s), and (B, Σ_b) be measurable spaces; let ν be a measure on $(S \times Q, \Sigma_s \times \Sigma_q)$; let $\lambda: Q \times \Sigma_b \to$ extended reals be an abcont conditional measure.

Then, for any measurable subset G of $(S \times B, \Sigma_s \times \Sigma_b)$, the integral

2pt brackets

90 36 230 126

$$\int_{S\times Q} \lambda\Big(q, \{b \mid (s,b) \in G\}\Big)\, \nu(ds, dq) = \mu(G) \qquad (4.5.1)\ \cancel{(1)}$$

9.12.12

is well-defined, and the resulting set function μ is a measure on $(S \times B, \Sigma_s \times \Sigma_b)$.

Proof: We show that the conditions for the existence of a product measure on the space $(S \times Q \times B, \Sigma_s \times \Sigma_q \times \Sigma_b)$ are satisfied. We ~~shall~~ consider this as a product of two spaces, the first being $(S \times Q, \Sigma_s \times \Sigma_q)$ and the second being (B, Σ_b). ν is a measure over the first space.

Define $\lambda': S \times Q \times \Sigma_b \to$ extended reals by the rule

$$\lambda'(s, q, E) = \lambda(q, E),$$

all $\underline{s} \in S$, $\underline{q} \in Q$, $\underline{E} \in \Sigma_b$. One easily verifies that λ' is abcont conditional. Hence μ', defined by

$$\mu'(\underline{H}) = \int_{S\times Q} \lambda'\big(\underline{s},\underline{q},\{\underline{b} \mid (\underline{s},\underline{q},\underline{b}) \in \underline{H}\}\big)\nu(\underline{ds},\underline{dq}) \qquad (2)$$

(4.5.2)

for any $\underline{H} \in \Sigma_s \times \Sigma_q \times \Sigma_b$, is a measure on $(\underline{S} \times \underline{Q} \times \underline{B})$. However, μ in (1) ~~, however,~~ is merely the marginal of this measure on the component space $(\underline{S} \times \underline{B}, \Sigma_s \times \Sigma_b)$; i.e., ~~That is,~~ $\mu(\underline{G}) = \mu'(\underline{G} \times \underline{Q})$ for any measurable $\underline{G} \subseteq (\underline{S} \times \underline{B})$. This may be verified by substituting $\underline{G} \times \underline{Q}$ for $\underline{H}$ in (2); simplification yields (1). Hence μ itself is a measure.

Let us now interpret this theorem concretely. $\underline{S}$, $\underline{Q}$, and ν have the meanings already discussed. Let $(\underline{B},\Sigma_b) = (\underline{R} \times \underline{T}, \Sigma_r \times \Sigma_t)$; then we may interpret λ to be the family of production measures λ_1, so that $\lambda(\underline{q},\cdot) = \lambda_1(\underline{q},\cdot)$ is the production measure on $\underline{R} \times \underline{T}$ associated with the activity $\underline{q}$.

μ in (1) now becomes a measure on $(\underline{R} \times \underline{S} \times \underline{T}, \Sigma_r \times \Sigma_s \times \Sigma_t)$.[11] How is this to be interpreted? Contemplation of (1), and the nature of ν and λ, suggests that μ is the <u>total production measure resulting from the activity assignment</u> ν. Thus, if we take any region $\underline{F}$, and consider for each activity the mass produced of resource-types $\underline{E}$ in time-period $\underline{G}$, then $\mu(\underline{E} \times \underline{F} \times \underline{G})$ is the limit of the weighted sum of these masses, the weights being provided by the assignment ν restricted to $\underline{F} \times \underline{Q}$.

Precisely the same interpretation, but with λ_2 instead of λ_1, now yields a measure μ on $\underline{R} \times \underline{S} \times \underline{T}$ that ~~which~~ is to be interpreted as the total consumption measure resulting from the assignment ν.

Finally, ~~let us~~ we interpret (B,Σ_b) to be the space of transmutation-paths (Ω_r,Σ') and λ to be the family of measures ρ on this space. ~~That is~~ i.e., $\lambda(q,\cdot) = \rho(q,\cdot)$ is the "capital-goods" measure associated with the activity q.

raised dot

With this interpretation, μ in (1) becomes a measure on the space $(S \times \Omega_r, \Sigma_s \times \Sigma')$. How is this to be interpreted? First ~~of all~~, consider the set $S \times \Omega_r$. A moment's reflection shows ~~that~~ this can be identified with the set of all histories ~~which~~ that have constant itineraries. ~~let us~~ We call these the sedentary histories. ~~For~~ the point $(s_o,h_r) \in S \times \Omega_r$ corresponds naturally to the history h whose transmutation-path is h_r, and whose itinerary has the constant value s_o over the time-interval in which it exists. The natural interpretation of μ here is as the distribution of mass over the space of sedentary histories. Thus, letting F be a region and H a measurable set of transmutation-paths, $\mu(F \times H)$ equals the total mass embodied in sedentary histories ~~which~~ that are located in F and have transmutation-paths in H.

(If, at the cost of further complications, we had introduced activities involving non-sedentary histories, the μ here would come out to be the basic world-description measure over the space of all histories, Ω.)

To summarize: Any activity assignment ν determines a triple of measures, all ~~via formula~~ by (1). When we substitute for λ in (1) the production and consumption measures, λ_1 and λ_2, respectively, of the various activities in Q, we come out with the production and consumption measures over $R \times S \times T$ ~~which~~ that are yielded by this assignment ν. When we substitute the "capital-goods" measures ρ of the various activities for λ, we come out with the mass distribution over sedentary histories.

We ~~shall~~ abbreviate these three measures as μ_1, μ_2, and μ_θ, respectively. Thus μ_1 and μ_2 are on the space $(R \times S \times T, \Sigma_r \times \Sigma_s \times \Sigma_t)$, while μ_θ is on the space $(S \times \Omega_r, \Sigma_s \times \Sigma')$.

Before going on to discuss the feasibility tests ~~which~~ arising from this analysis, let us see what it reduces to in a very simple case: ~~the case~~ where all four sets, Q, R, S, T, are finite (and all subsets are measurable). This case is of interest for two reasons. First, it gives a heuristic guideline to the analysis just completed. Second, it shows how everything boils down to what is essentially ordinary activity analysis.

9.12.13

The value of assignment ν at the singleton set $\{(s,q)\}$ will be written simply as ν_{sq}, and we adopt a similar notation for all other measures. In fact, in this simple case the measures can be thought of as ordinary point functions, and the notation underlines this fact; ν_{sq} is the "level" at which activity q is running at location s.

For convenience we let λ stand for either λ_1 or λ_2. Then λ_{qrt} equals total production (or consumption) of resource r at time t in activity q. In (1) ~~let us~~ we choose for G the singleton set $\{(r,s,t)\}$. Then the integral (1) reduces to a simple summation over activities $q \in Q$:

$$\mu_{rst} = \nu_{sq_1}\lambda_{q_1rt} + \nu_{sq_2}\lambda_{q_2rt} + \ldots .$$

In this case the interpretation of μ is obvious: the total production (or consumption) of resource r at location s at time t, obtained by taking a weighted sum of production (consumption) of r at t for each allowable activity, the weights being the levels of the various activities at s, as indicated by assignment ν.

Things are slightly more complicated for the "capital-goods" measure ρ. For simplicity ~~let us~~ we ignore "births" and "deaths", and assume that all transmutation paths exist at all times. If N is the number of time-points, ~~then~~ a transmutation-path may be written as an N-tuple $(r_1,\ldots,r_N)$ in R, r_t being its resource-state at time t. Ω_r may then be identified with the product space R^N, and ~~Then~~ $\rho_{q,r_1,\ldots,r_N}$ is the mass embodied in the transmutation-path $(r_1,\ldots,r_N)$ in activity q. Now ~~let us~~ we choose for G in (1) the singleton $\{(s,r_1,\ldots,r_N)\}$. The integral (1) again reduces to a summation over activities $q \in Q$:

$$\mu_{s,r_1,\ldots,r_N} = \nu_{sq_1}\rho_{q_1,r_1,\ldots,r_N} + \nu_{sq_2}\rho_{q_2,r_1,\ldots,r_N} + \ldots .$$

bl left
Then

$\mu_{\underline{s},\underline{r}_1,\ldots,\underline{r}_N}$ is the total mass embodied in the sedentary history, whose location is fixed at $\underline{s}$, and which runs through the sequence of resource-states $(\underline{r}_1,\ldots,\underline{r}_N)$ over ~~Time~~. This again is a weighted sum of the mass embodied in transmutation-path $(\underline{r}_1,\ldots,\underline{r}_N)$ for each allowable activity, the weights being the levels of the various activities at $\underline{s}$.

~~Let us~~ We now return to the general case. So far we have said nothing about feasibility, but ~~We~~ ~~now~~ propose a test ~~which~~ that has a vague resemblance to the configuration test (loose constructionist version). A measure passes that test if it is a countable patching of allowable configurations. Here we have a corresponding set of allowable activities, $\mathcal{Q}$, and we consider only measures ~~which~~ that can be "built up" from the activities of $\mathcal{Q}$. We interpret "built up" to mean that there ~~exists~~ is an assignment ν such that the measure μ to be tested is determined by ν according to (1). That is, considering (1) as a function ~~which~~ that assigns a measure μ to every measure ν, the measures ~~which~~ passing this test are those in the range of the function.

This might be considered too easy a test, since ν is not restricted in any way. One natural restriction that might be placed on ν is that it be ~~sigma~~ σ-finite or even bounded.

Another constraint on ν that suggests itself is an areal capacity restriction. We have ~~already~~ discussed this in connection with cross-sectional constraints, where we postulated a "demand-for-space" function $\underline{f}: \underline{R} \times \underline{S} \rightarrow$ reals, which restricted the possible cross-sections $\mu_{\underline{t}}$ at moment $\underline{t}$. Now the measure μ_0 (zero) on the space of sedentary histories, $S \times \Omega_r$, determines a cross-sectional measure μ_t for every moment t, hence must satisfy an areal capacity contraint for all t. ~~In fact~~ we must have

$$\int_{F\times\Omega_r} f(h_r(t),s)\,\mu_0(ds,dh_r) \leq \alpha(F)$$

for all regions F and all moments t, where α is ideal area. This is essentially a restatement of (3.3) ~~(3) of section 3 above.~~

It could be argued, however, that this understates the demand for space. An activity not only needs space to store its "capital-goods" at any moment, but also "aisle space" or "elbow room", i.e., enough extra vacant area ~~in short~~ to carry out the manipulations and transformations in which it is involved. This suggests ~~that one~~ we should attach a "demand-for-space" function directly to activities *per se*. Thus ~~let us~~ we write f: $S \times Q \to$ reals, f non-negative, measurable with respect to

9.R.14 $\Sigma_s \times \Sigma_q$. $f(s,q)$ is to be understood intuitively as the space demand by (unit level of) activity q at location s. We must satisfy

$$\int_{F\times Q} f\,d\nu \leq \alpha(F), \qquad (4.5.3)\ ~~(3)~~$$

for all regions F. This places a direct restriction on the possible assignments ν, and therefore a further indirect restriction on the measures μ_0, μ_1, μ_2 ~~which~~ that must satisfy (1) from some ν.

We now turn to the individual measures μ_0, μ_1, μ_2. Our ultimate aim is to establish feasibility conditions for measures μ on the space of histories Ω. How does such a μ relate to this triple of measures? As for μ_0, its universe set is the set of sedentary histories, $(\underline{S} \times \Omega_r)$, which is a subset of Ω. The feasibility condition on μ, then, is that its restriction to $\underline{S} \times \Omega_r$ be an allowable μ_0.[12]

As for the production and consumption measures, μ_1 and μ_2, these represent "births" and "deaths" of histories, respectively. Thus, a finished product is produced in a manufacturing process at a certain place and time. This initiates a new history, which perhaps moves into transportation to be consumed elsewhere. The smoke emitted from a chimney initiates histories involved in atmospheric circulation. We must have, ~~then,~~ for measurable $(\underline{E} \times \underline{F} \times \underline{G}) \subseteq (\underline{R} \times \underline{S} \times \underline{T})$, $\mu_1(\underline{E} \times \underline{F} \times \underline{G})$ equal to the total mass embodied in histories ~~which~~ that originate in period $\underline{G}$ in region $\underline{F}$ at a resource-type in $\underline{E}$. A similar relation holds for μ_2 and the mass of histories ~~which~~ terminating in $\underline{E} \times \underline{F} \times \underline{G}$.[13]

For μ to pass the feasibility test, ~~then,~~ there must be an assignment ν yielding measures μ_0, μ_1, μ_2 which are <u>simultaneously</u> compatible with μ.

4.6. Neighborhood Effects

We now consider some of the presuppositions implicit in the preceding construction. These are~~, in fact,~~ worth studying on their own, and not merely in connection with the activity analysis model.

Let F and G be two disjoint regions. ~~It can happen that~~ The possible processes ~~which~~ that can go on in region G ~~are~~ can be influenced by what goes on in region F. For example, the sound, light, heat, or substances emanating from F may condition the environment of G. These influences are sometimes called <u>neighborhood effects</u>. We may expect, in a general way, that neighborhood effects will become stronger the closer F and G are to each other, while they tend to disappear between distant regions. (Some influences, ~~such as~~ e.g., radioactive fallout, have worldwide effects. If we include in the concept of neighborhood effect the deliberate propagation of influence via the transport-communications system, in addition to the "natural" influences just mentioned, ~~then~~ even distant regions will be palpably influenced by each other.)

Now consider the opposite case, where there are no neighborhood effects. This may be assumed as a simplifying approximation when influences are sufficiently weak. But how exactly does one formulate the concept: "no neighborhood effects occur"? ~~Our~~ The next few paragraphs ~~represent an~~ attempt to pin down this notion.

The concept of direct summation of measure spaces has already been considered as a special case of "patching" (~~page)~~ Section 2.4). Explicitly, we have the following definition:

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Definition: Let (A_n, Σ_n, μ_n), $n = 1, 2, \ldots$, be a countable collection of measure spaces, where the A_n's form a packing: $A_m \cap A_n = \emptyset$ if $m \neq n$. The direct sum of these spaces is the triple (A, Σ, μ), where

(i) $A = A_1 \cup A_2 \cup \ldots$;

(ii) Σ consists of all sets of the form $E_1 \cup E_2 \cup \ldots$, where $E_n \in \Sigma_n$ for all $n = 1, 2, \ldots$;

(iii) μ has domain Σ, and, for $E = E_1 \cup E_2 \cup \ldots$, $E_n \in \Sigma_n$,

$$\mu(E) = \mu_1(E_1) + \mu_2(E_2) + \ldots . \qquad \text{(4.6.1)} \; \cancel{(1)}$$

With this definition, (A, Σ, μ) is a measure space. It is not difficult to show that Σ is closed under countable unions, and differences, and that $A \in \Sigma$, so that Σ is a ~~sigma~~ σ-field with universe set A. For each n, Σ_n is the restriction of Σ to subsets of A_n. Finally, the disjointness of the A_n's guarantees that, for any $E \in \Sigma$, its representation in the form $E_1 \cup E_2 \cup \ldots$ is unique. Then μ as given by (1) is well-defined and μ_n is the restriction of μ to A_n. The fact that μ is a measure is a simple consequence of the patching theorem.

We shall use the symbol $\oplus$ for direct sums. Thus, $\Sigma = \Sigma_1 \oplus \Sigma_2 \oplus \ldots$, and $\mu = \mu_1 \oplus \mu_2 \oplus \ldots$.

Next, we want to extend this definition to the case where there is a whole set of measures, $\mathfrak{M}_n$, defined on each space (A_n, Σ_n), not merely the single measure μ_n. Write $(A_n, \Sigma_n, \mathfrak{M}_n)$ for the measurable space together with the set of measures. Again we assume that the A_n's form a packing.

Definition: The direct sum $(A_1, \Sigma_1, \mathcal{M}_1) \oplus (A_2, \Sigma_2, \mathcal{M}_2) \oplus \ldots$ is the triple $(A, \Sigma, \mathcal{M})$, where A and Σ are formed as above, and $\mathcal{M}$ is the set of all measures μ formed according to (1), where $\mu_1, \mu_2, \ldots$ are selected from $\mathcal{M}_1, \mathcal{M}_2, \ldots$ in all possible ways.

Now let $(S \times A, \Sigma_s \times \Sigma_a)$ be a product measurable space. (We shall later interpret S to be physical Space, but for the time being let us proceed abstractly.) Let $\mathcal{M}$ be a set of measures on this space.

Definition: $(S \times A, \Sigma_s \times \Sigma_a, \mathcal{M})$ is countably rectangular iff, for any countable measurable partition $\{S_1, S_2, \ldots\}$ of S, it is the direct sum

$$(S_1 \times A, \Sigma_{s_1} \times \Sigma_a, \mathcal{M}_1) \oplus (S_2 \times A, \Sigma_{s_2} \times \Sigma_a, \mathcal{M}_2) \oplus \ldots .$$

Here Σ_{s_n} is the restriction of Σ_s to subsets of S_n, and $\mathcal{M}_n$ is the set of all restrictions to S_n of the measures $\mu \in \mathcal{M}$.

Here are some examples:

1. ~~(i)~~ Let S consist of two points, A of one point: $S = \{s_1, s_2\}$, $A = \{a\}$. All subsets are measurable. Let $\mathcal{M}$ consist of the four measures μ_{ij} ($i = 1, 2$; $j = 1, 2$) whose values on the two points of $S \times A$ are given by $\mu_{ij}\{(s_k, a)\} = i$ if $k = 1$, and $= j$ if $k = 2$. Then $\mathcal{M}$ is countably rectangular. But if any one of these measures is deleted, the remaining trio is not countably rectangular.

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2○ ~~(ii)~~ The set of all measures, and the set of all ~~sigma~~ σ-finite measures, on $S \times A$, are both countably rectangular.

3○ ~~(iii)~~ The set of all bounded measures is not countably rectangular, if the ~~sigma~~ σ-field Σ_s is infinite. (The proofs of these statements are left as exercises.)

~~The idea is that,~~ Under countable rectangularity, the set $\mathcal{M}$ is built up from component sets in roughly the same way ~~that~~ a rectangle set in a cartesian product space is built up from the "sides" of the rectangle.

Now let us interpret S as Space. The set A will be given a variety of interpretations, but in all cases the set $\mathcal{M}$ will be some "allowable" set of measures.

This apparatus is designed to capture the intuitive notion that, if there are no neighborhood effects, ~~then~~ any region can be "autonomously" assigned its own allowable set of measures, this set not depending at all on what is chosen elsewhere. Collection $\mathcal{M}_n$ plays the role of this autonomous set for the region S_n, and the "countable rectangularity" property expresses precisely the fact that the choices from the respective sets $\mathcal{M}_n$ can be made freely and independently of each other.

Fig 13-2.

We now ~~Now let us~~ take (A, Σ_a) to be the space of allowable activities (Q, Σ_q), and take $\mathcal{M}$ to be the allowable assignments ν on $S \times Q$. Then in general this will be countably rectangular in the activity analysis setup. We have already noted this if the assignment ν can be any measure, or any ~~sigma~~ σ-finite measure.

Slightly less obvious is the fact that, even if a space-capacity constraint of the form (5.3) ~~(3) of section 5~~ (repeated as (2) below) is imposed, the resulting set of allowable assignments retains this following property:

Theorem: Let $(\underline{S},\Sigma_s)$ and $(\underline{Q},\Sigma_q)$ be measurable spaces, $f: \underline{S}\times\underline{Q} \to$ reals non-negative and measurable, α a measure on $\underline{S}$. Then the set of measures ν on $\underline{S}\times\underline{Q}$ that ~~which~~ satisfy

$$\int_{F\times Q} f\, d\nu \le \alpha(F), \qquad (4.6.2)\ \cancel{(2)}$$

for all measurable $F \subseteq \underline{S}$, is countably rectangular.

Proof: Let $\{\underline{S}_1, \underline{S}_2, \ldots\}$ be a countable measurable partition of $\underline{S}$. For any ν satisfying (2), ν_n, its restriction to $\underline{S}_n \times \underline{Q}$, satisfies the same condition for all measurable $F \subseteq \underline{S}_n$. Now let ν_n be such a measure on $\underline{S}_n \times \underline{Q}$, $n = 1, 2, \ldots$, and consider the direct sum $\nu' = \nu_1 \oplus \nu_2 \oplus \ldots$. For any measurable $F \subseteq \underline{S}$, we have

$$\int_{F\times Q} f\, d\nu' = \int_{(F\cap S_1)\times Q} f\, d\nu' + \int_{(F\cap S_2)\times Q} f\, d\nu' + \ldots$$

$$= \int_{(F\cap S_1)\times Q} f\, d\nu_1 + \int_{(F\cap S_2)\times Q} f\, d\nu_2 + \ldots \le \alpha(F\cap S_1) + \alpha(F\cap S_2) + \ldots = \alpha(F).$$

break at = or ≤

(F∩–)×Q all subscripts

Thus ν' satisfies (2). This proves countable rectangularity. |||

Next, ~~let us~~ we turn to the sets of allowable measures μ_0, μ_1, μ_2 determined by the allowable assignments ~~via (1) of section 5~~ of (5.1). The following result applies to all three ~~of these~~ measures.

Theorem: Let measure μ on $\underline{S} \times \underline{B}$ be determined by ν on $\underline{S} \times \underline{Q}$ by the rule

$$\mu(\underline{G}) = \int_{\underline{S}\times\underline{Q}} \lambda\Big(\underline{q}, \{\underline{b} \,|\, (\underline{s},\underline{b}) \in \underline{G}\}\Big)\nu(\underline{ds},\underline{dq}), \qquad (4.6.3)$$

where $\underline{G}$ is a measurable subset of $\underline{S} \times \underline{B}$, and $\lambda: \underline{Q} \times \Sigma_{\underline{b}} \to$ extended reals is an abcont conditional measure. Then, if the set of allowable measures ν is countably rectangular, the same is true for the resulting set of allowable measures μ.

Proof: Let $\{\underline{S}_1, \underline{S}_2, \ldots\}$ be a countable measurable partition of $\underline{S}$. If μ satisfies (3) for some allowable ν, its restriction to $\underline{S}_n \times \underline{B}$ satisfies (3) for all measurable $\underline{G} \subseteq \underline{S}_n \times \underline{B}$, with $\underline{S}_n$ substituted for $\underline{S}$, and ν_n, the restriction of ν to $\underline{S}_n \times \underline{Q}$, substituted for ν. Now let μ_n be such a measure on $\underline{S}_n \times \underline{B}$, determined by ν_n on $\underline{S}_n \times \underline{Q}$, $\underline{n} = 1, 2, \ldots$. Consider the direct sums $\mu' = \mu_1 \oplus \mu_2 \oplus \ldots$, and $\nu' = \nu_1 \oplus \nu_2 \oplus \ldots$. By the countable rectangularity assumption, ν' is allowable.

For any measurable $\underline{G} \subseteq \underline{S} \times \underline{B}$, we have

$$\mu'(\underline{G}) = \mu'\Big(\underline{G}\cap(\underline{S}_1\times\underline{B})\Big) + \mu'\Big(\underline{G}\cap(\underline{S}_2\times\underline{B})\Big) + \ldots$$
$$= \mu_1\Big(\underline{G}\cap(\underline{S}_1\times\underline{B})\Big) + \mu_2\Big(\underline{G}\cap(\underline{S}_2\times\underline{B})\Big) + \ldots$$

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$$= \int_{S_1\times Q} \lambda\left(q, \{b \mid (s,b) \in G\}\right) \nu_1(ds,dq) + \dots$$

$$= \int_{S\times Q} \lambda\left(q, \{b \mid (s,b) \in G\}\right) \nu'(ds,dq).$$

Hence μ' is allowable. Thus the set of allowable measures μ is countably rectangular. ∎

The assumption of no neighborhood effects, therefore, pervades the activity analysis model we have constructed. How realistic is this? There will ~~of course~~ always be some neighborhood effects, so the real question is whether these effects are unimportant enough to be ignored. The answer appears to depend on the scale of observation. On a "person-sized" level, neighborhood effects are so vital that any model ignoring them would be useless. Chopping a person in half will rapidly affect his functioning: each half needs the "neighborhood effects" emanating from the other. Similarly, the technical possibilities in half a machine or half a house will be affected if the other half is sheared off.

At the level of the ordinary urban neighborhood, the neighborhood effects are still important but not nearly as momentous. "Urban problems" are in large measure the reflection of these interdependencies, resulting from the proximity of masses of people to each other. Going up the scale to the economy-wide and world-wide levels, neighborhood effects are much attenuated. We might expect, then, that the activity analysis model described here could be fairly applicable to economy-wide technical possibilities, would be less so at the urban neighborhood level, and would be poor as a model for individual household possibilities.

There is~~, however,~~ one consideration ~~which~~ that vitiates conclusions of this sort. It is the system of constraints as a whole ~~which~~ that is subject to criticism, not any particular subsystem in isolation. If neighborhood effects are not taken into account in the activity analysis subsystem, they may be taken account of elsewhere, in such a way that the set of measures passing all feasibility tests reflects the existence of these effects.

~~There are~~ other "neighborhood effect" concepts ~~which~~ are not captured by the countable ~~regularity~~ rectangularity property. ~~One~~ We sometimes want~~s~~ an asymmetrical concept, in which region E has an effect on region F but not vice versa. Countable ~~regularity~~ rectangularity is symmetric, in the sense that no ordering distinctions are made among the components of a direct sum. We ~~shall very~~ briefly indicate how these "one-way" effects might be represented. This is done by bringing in Time explicitly.

First we need a slight weakening of the countable rectangularity concept. Let $\underset{\sim}{M}$ be a set of measures on space $(\underline{A},\Sigma)$, and let $\underline{E}$, $\underline{F}$ be two disjoint measurable subsets of $\underline{A}$. Then $\underset{\sim}{M}$ is said to be rectangular with respect to the pair of sets $\underline{E}$, $\underline{F}$ iff

M_{EUF}

$$\underset{\sim}{M}_{E\cup F} = \underset{\sim}{M}_E \oplus \underset{\sim}{M}_F, \qquad \text{(4.6.4)} \ ~~(4)~~$$

where M_E is the set of restrictions to E of the measures $\mu \in M$, and similarly for M_F and $M_{E \cup F}$. (Countable rectangularity implies that (4) is true for any such E, F).

Now let M be the set of allowable measures on the space $(B \times S \times T, \Sigma_b \times \Sigma_s \times \Sigma_t)$. Here S and T are Space and Time; B might be the resource set, R, or some more complicated set, depending on the problem in hand. Let $F_1, F_2 \subseteq S$ be two disjoint regions.

Now we define: There are no neighborhood effects from F_1 to F_2 across time-instant t_0 iff M is rectangular with respect to the two sets

$$\left(B \times F_1 \times \{t|t < t_0\}\right) \quad \text{and} \quad \left(B \times F_2 \times \{t|t > t_0\}\right).$$

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This amounts to saying that what can happen on F_2 after time t_0 is not affected by what happens on F_1 prior to t_0. This concept is not symmetrical, ~~in the sense that~~ since there can be neighborhood effects across t_0 from F_2 to F_1, but not from F_1 to F_2.

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4.7. Superposition and Returns to Scale

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Definition: Let M be a non-empty set of measures on space (A, Σ). Set M is said to be additive iff, whenever μ_1 and μ_2 belong to M, then $\mu_1 + \mu_2$ belongs to M, and M is said to be a cone iff, whenever $\mu \in M$ and $c \geq 0$ is a real number, then $c\mu \in M$.

An additive set of measures is also said to obey the superposition principle, since two members of it may be "superimposed" to form a third member. ~~Note that~~ a non-empty M that ~~which~~ is both additive and conical is a convex cone.[14]

The set of all measures, of all σ-~~sigma~~-finite measures, and of all bounded measures on $(\underline{A},\Sigma)$ are examples of sets having both ~~of~~ these properties. Now consider the set of all allowable assignments ν on the space $\underline{S} \times \underline{Q}$ in the activity analysis model. This determines a set of measures μ on the space $\underline{S} \times \underline{B}$, where $\underline{B}$ and μ have various interpretations, by ~~via~~ the integration formula (5.1) ~~(1) of section (5)~~.

It follows at once from elementary integration theorems that if ν' determines μ', and ν'' determines μ'' via (5.1) ~~(1)~~, then $\nu' + \nu''$ determines $\mu' + \mu''$. Hence if the set of allowable assignments ν is additive, so is the resulting set of measures μ. Similarly, if the set of allowable ν's is a cone, so is the resulting set of allowable μ's.

Are these conditions realistic? As ~~was~~ noted ~~above~~ in the discussion of neighborhood effects, any conclusion is to be treated with caution: Even if we decide (as we shall) that these conditions are not very defensible, it still does not follow that we ~~one~~ should reject feasibility tests that ~~which~~ assume them. The system of feasibility tests as a whole must be confronted. As a simple example, consider the activity analysis model in which the set of allowable assignments is unrestricted. This allows, say, cross-sectional measures of arbitrarily high density, which is not realistic. But there are other feasibility tests that ~~which~~ exclude excessive densities — in particular, space capacity constraints. It may be very convenient to keep the unrestricted activity analysis model as one subsystem of constraints, and no objections need arise to the system of constraints as a whole.

With this caution in mind, ~~let us~~ we pose the question in the following form. Given the set of allowable configuration-types or activity types, is it reasonable to suppose that this set is additive and/or conical?

There is one minor difficulty involved in the concept of additivity here, which we illustrate with the set of allowable configuration-types. A configuration-type is a measure on a universe set of the form $R \times F$, where F is an "abstract" region ~~which~~ that is a measurable and a metric space. Now consider μ_1 and μ_2, defined on $R \times F_1$, $R \times F_2$, respectively. Since in general F_1 and F_2 are not the same, the sum $\mu_1 + \mu_2$ is not well-defined. We can, however, proceed as follows. Suppose there is a measurability-preserving isometry between F_1 and F_2, say $f: F_1 \to F_2$. This, with μ_1, induces a measure μ_1' on $R \times F_2$; and we now define the sum of μ_1 and μ_2 as $\mu_1' + \mu_2$, a measure on $R \times F_2$. If there are several different isometries between F_1 and F_2, in general each will lead to a different summation operation. If there is no isometry, ~~then~~ the sum is not defined. These complications reduce the usefulness of the additivity concept in this context.

By contrast, the condition that the set of allowable configuration-types or activity-types is a cone is perfectly well-defined. For configurations, this reads: If μ on $R \times F$ is an allowable configuration-type, ~~then~~ so is $k\mu$ on $R \times F$, k

being a non-negative real number. We ~~shall~~ devote most ~~the bulk~~ of our attention to the question of reasonableness of this condition, and the corresponding condition for activity-types.

Let us connect this with the concept of scale. Recall that in our discussion of scale (~~in~~ Section 2.7) we distinguished a number of different concepts, in particular the notions of a k-fold expansion in the intensive, the extensive, and the duplicative senses. Now suppose ~~that~~ the set $\mathcal{M}$ of allowable configuration-types, or activity-types, has the following property: if $\mu \in \mathcal{M}$ and μ' is the k-fold expansion of μ in the x-sense, ~~then~~ $\mu' \in \mathcal{M}$. In this case we say that $\mathcal{M}$ has constant returns to scale in the x-sense. We ~~shall~~ discuss each of the various senses in turn.

A moment's reflection shows that the two conditions "$\mathcal{M}$ is a cone", and "$\mathcal{M}$ has constant returns to scale in the intensive sense", are the same. How reasonable is this property? That is, if μ is allowable, is it reasonable that $k\mu$ should be allowable, for any real number $k \geq 0$?

There are two cases. (i) For $k > 1$, arbitrarily high densities would be allowable. But presumably at some point it would become physically impossible to squeeze that mass into the given space; and even before one reaches this density, the increasing concentration of mass will in general lead to interactions (neighborhood effects!) that ~~which~~ prevent an exact proportional change of mass everywhere. (ii) For $k < 1$, this last objection still holds, in reverse. There may be "threshold effects" or "critical masses" that ~~which~~ prevent one from halving mass everywhere and maintaining feasibility.

13-5 ~~Note th~~at when $k \geq 0$ is an integer, and $\mu \in M$, then $k\mu \in M$ follows both from the condition that M is a cone, and from the condition that M is additive. Thus the objections against arbitrarily high k values are objections against both ~~of~~ these conditions.

Now consider constant returns to scale in the extensive sense. start Here ~~one~~ we introduce~~s~~ the areal measure α on Space, and a k-fold expansion involves a similarity mapping ~~which~~ that multiplies area, as well as all masses, by k. All densities (with respect to α) remain the same, but now another difficulty arises. If volume expands by a factor of k, ~~then~~ surface area expands by $k^{2/3}$ and length by $k^{1/3}$. These non-proportional changes in general make it impossible to maintain an extensive scale change. For example, if a house is doubled in linear dimension, its "capacity" (roughly measured by volume) octuples, but the rate of heat loss (roughly proportional to surface area) only quadruples, so the heating plant need not expand in proportion. Absolute scale changes do make a difference, and constant returns to scale in the extensive sense must also be rejected
15 as a general rule.[15]

This brings us to constant returns to scale in the duplicative sense. Here an activity type or configuration-type is placed "side-by-side" with itself. Specifically, if μ is an allowable configuration, with universe set $R \times F$, ~~then~~ a k-fold expansion of μ is a measure μ' on $R \times F'$ such that there is a partition $\{F_1', F_2' .., F_k'\}$ of F' into k pieces, and μ' restricted to each piece $R \times F_i'$ $(i = 1,...,k)$ is a duplicate of μ (i.e. ~~that is~~, there is a measurability-preserving isometry f from F to F_i', and μ' restricted is the measure induced by f from μ). A similar definition holds for activity-types.

~~We~~ note, first ~~of all,~~ that k must be an integer for this definition to be meaningful. A hen and a half does not lay an egg and a half if one hen lays one egg. This gives at best a weaker condition than constant returns *per se*, and might be called constant *integer* returns to scale.

~~Note~~ also, ~~that,~~ unlike the other scale concepts, ~~there are~~ many *distinct* configuration-types ~~which~~ are k-fold expansions of the same μ. because ~~The reason is that~~ nothing is said about the metric relations of the pieces F_i' to each other, but only about their internal structure. The k pieces may be close to each other or scattered. Under constant duplicative returns, all ~~of~~ these k-fold expansions would be allowable.

Constant duplicative returns is a corollary of one of the feasibility test systems we have discussed: the configuration-criterion, in the loose constructionist version. ~~For~~ a cross-sectional measure passes this test iff it is a (countable) patching of exemplifications of allowable configuration-types. If μ is allowable, ~~then~~ a k-fold expansion is such a patching (in fact it is a direct sum of the k-duplicates). A similar statement holds for activities in place of configurations.

The basic weakness of the constant duplicative returns assumption is that shared by this loose constructionist version: the ignoring of neighborhood effects. In general, what is feasible in a region depends on the environment of that region, and cannot simply be drawn from a fixed list of allowable possibilities. However, it may be a fair approximation in some situations, and as such it appears to be the least objectionable of the three senses of "constant returns to scale" ~~that~~ we have discussed.

A

4.8. Indivisibility

A long tradition in economic theory connects departures from "constant returns to scale" with "indivisibility". Another, somewhat more recent, literature denies the connection. Our aim here is not to resolve this issue once and for all, but to clarify it by distinguishing the many different concepts named by these terms. It is likely that much of the controversy arises from the confusion of meanings of the same term in the minds of different participants (or of the same participant). In ~~the preceding~~ section 4.7 we have distinguished several different meanings of "constant returns to scale". In the ~~present~~ section we ~~shall~~ do the same for "indivisibility."[16]

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One may distinguish at least six different meanings of the term "indivisibility" (many of ~~these~~ which have already been discussed):

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~~(i)~~ 1. As a synonym for integer-valuedness.

~~(ii)~~ 2. As the requirement that a certain measure be finitely concentrated, in particular, that it be simply-concentrated. The man who "flung himself upon his horse, and rode madly off in all directions" was violating an indivisibility constraint in this sense.[17]

~~(iii)~~ 3. As a limited variety constraint. This may be due to natural laws, as in biology, where only a limited range of organic forms are viable; ~~or~~ it may be due to lack of knowledge of how to make certain resource-types or configurations; or it may be due to a high overhead cost of starting new product lines, or to lack of raw materials.

~~(iv)~~ 4. As the condition that certain configurations cannot be split into two spatially separated halves. This is of course the original meaning of the term "indivisibility".

~~(v)~~ 5. As the condition that certain configurations cannot be split without destroying their functioning, as with "organic wholes".

~~(vi)~~ 6. As the condition that certain configurations cannot be spatially segregated by resource components; e.g. a metal ~~which~~ that cannot be extracted from its ore.

Some of these spatial interpretations have temporal equivalents, e.g. "non-interruptibility" constraints.

~~Now,~~ surveying these interpretations, and referring back to our critical discussion of returns to scale, ~~one~~ we notes that none of these conditions was used in the argument. Constant intensive returns was rejected because of threshold and congestion effects, constant extensive returns because of length-area-volume non-proportionalities, and constant duplicative returns because of neighborhood effects. (Threshold, congestion, and length-area-volume phenomena can themselves probably be regarded as special manifestations of neighborhood effects). Thus it would appear that non-constant returns to scale (in any sense) can appear without indivisibilities (in any sense of the term).

In fact, one might be well-advised to reverse the standard argument and derive certain kinds of indivisibility conditions from the non-constancy of returns to scale. Suppose length-area-volume effects make an organic form viable only in a limited size range. This is an example of non-constant extensive returns, and leads to an indivisibility of type ~~(iii)~~ 3.

The existence of neighborhood effects underlies indivisibilities of type ~~(v)~~ 5, and perhaps also of types ~~(iv)~~ 4 and ~~(vi)~~ 6.

4.9. Spatial Control

Most of our discussion has been of tests ~~which~~ That any feasible measure μ on the space of histories Ω must satisfy. In this section we take a different point of view and consider some means by which the acting person carries out his choice among the feasible alternatives. The discussion will be entirely informal.

We ~~shall be~~ are concerned with spatial control, ~~— that is~~ i.e., the control of the movements of things. It appears that spatial control underlies control in general[18] ~~(as was pointed out long ago by John Stuart Mill)~~. To explain: ~~In the first place,~~ ~~one~~ we ~~acts~~ act upon the world exclusively through motions of the body (including speech, which is a motion of the diaphragm, vocal cords, etc.). These acts influence objects or other people. ~~One gets~~ We cause things to interact, as a rule, by placing them in proximity. A plan of action may be in large part described as a schedule, bringing people and/or objects together at various points of Space-Time to interact in desired ways, the outputs of some of these processes being stored or transported to arrive at other Space-Time points where they serve as inputs for other processes. (These processes include not only "production" in the ordinary sense of the term, but residential processes, education, dances, political meetings, etc.)

For such a plan to be feasible, the transportation and storage facilities must be available when and where needed, and it must be possible to carry out the processes with the scheduled factors. This will generally involve persuading other people to cooperate ~~— say~~ e.g., by exhortation, or offers of services or money.

g13-1

This account omits one important aspect of spatial control. ~~One~~ We not only ~~has to~~ must move things, but also to prevent motion. Everyone is aware of the fact that transportation incurs a cost — ~~that is~~ i.e., precludes some alternative opportunities by requiring the sacrifice of resources, time, and effort. The prevention of movement also incurs a cost. Perhaps this is noted so rarely because its manifestations are too obvious to comment upon. A few examples ~~will~~ illustrate this point.

There is, first, ~~of all,~~ a need to maintain altitude. That is, if things are to be brought into spatial proximity ~~in order~~ to interact, they must be at about the same distance from sea level. In the presence of gravity, the means of accomplishing this in almost all cases is to provide a horizontal surface ~~which~~ That gives common support to the various interacting entities. The crust of the Earth is available for this purpose, but has certain drawbacks. ~~First,~~ It may depart too far from the horizontal, so that ~~one has to~~ we must incur costs, either to flatten it ~~out~~, or to prevent things from rolling downhill, or both. ~~Second,~~ It may not provide adequate support, as in marsh or swamp, not to mention open water, and so must be either reinforced or unused. Finally, there may not be enough of it in the right places, so that more surface ~~is~~ must be constructed, at considerable cost. The prime example of this is multiple-story structures, but also most furniture serves the function of providing extra horizontal surface where needed: beds, chairs, tables, shelves.

Second~~ly~~, ~~one~~ we not only ~~has to~~ must bring the right things together to interact, but ~~to~~ also must keep the wrong things away. We have already commented on the function of barriers in keeping out the weather, "unauthorized personnel", etc. The entire institution of private property may be construed as a system of selective barriers, denying access to all except those authorized by the owner of the property, or those having special access rights (easements, search warrants, etc.). Nor is this merely a "capitalistic" arrangement: The phenomenon of "too many cooks spoiling the broth" is a universal technological problem, requiring the limitation of access rights in any economic system.

It should be clear from these examples that the prevention of motion is as fundamental a task as the provision of transportation. There is a close analogy here between the Resources-set, R, and Space, S. ~~One~~ We devote~~s~~ effort not only to transforming things from less to more desirable resource-states, but also to maintaining things in their present state: to the prevention or slowing of depreciation. ~~Indeed,~~ much of our total effort is of this "treadmill" variety, merely stopping things from getting worse -- most eating, sleeping, exercise, medical care, haircuts, laundering.

Similarly, in Space ~~one~~ we not only tries to move~~s~~ things to better locations (transportation), but also ~~tries~~ to prevent or slow their moving to worse locations. This could be called location maintenance. We have already examined the special case of altitude maintenance, and ~~shall~~ now discuss others. We ~~shall~~ continue to use the term barrier as a general name for any mechanism or institution ~~which~~ that maintains location.

Consider the very general class of barriers ~~which~~ we may call walls. These prevent various resource-types from moving through the border of a certain region. They ~~These~~ may be classified according to the kinds of resources ~~which~~ they bar, and whether they function to keep things in the region, or out of the region, or both.

Thus, a country may bar immigration, ~~or~~ emigration, or both. Glass is a barrier to the passage of air, but lets light through. A locked door is permeable to someone with the key; it is a barrier to others.

Storage facilities and packaging in general are all walls in this sense (cans, sacks, silos, barns, crates). ~~etc.~~ These serve the double function of protecting the contents by barring contaminants, the weather, pilferers, etc. from entering, and ~~also~~ holding the contents in place by barring exit.

A special class of walls, ~~which~~ (including clothing, window-shades, and soundproofing), serves the function of insuring privacy, ~~- that is,~~ i.e., prevents the dissemination of certain light or sound patterns that ~~which~~ might be perceived by outsiders.

Brakes are barriers that ~~which~~ prevent or limit the mobility of specific things. These include anchors, hobbles, paperweights, and ball and chain, as well as ordinary vehicle brakes.

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Bindings are mechanisms that ~~which~~ prevent or limit the relative motion of different things. These need not be barriers as we have been using the term, since the entire configuration of things bound together can move as a group relative to the earth. In fact, brakes may be considered the special case of bindings in which the earth itself is one of the objects. Bindings include adhesives, bolts, nails, zippers, string; they may include also packaging and storage facilities insofar as they hold things within an integument. But typically ~~one~~ we ~~wants~~ want not merely to bring things together but to hold them at proper relative distances. This is done by using a structural frame, which is a rigid body or one with a limited number of degrees of freedom for motion (~~for example~~ e.g., buildings, and the metallic or wooden parts of machines); the skeleton plays a similar role in organisms. These again are forms of bindings.

~~A great deal of~~ Much effort goes into the design of properly selective barriers (~~that is~~ i.e., ~~barriers~~ those ~~which~~ preventing the passage of some things and not others) and of barriers ~~which~~ that can be controlled to vary their selective power as desired. This involves both technological research and institutional arrangements (guards, customs inspectors, censors, ~~etc.~~). The evolution of military technology is to an extent a race between ever more penetrating offensive weapons and the finding of barriers to stop them, from the sword and shield to the missile

19 and antimissile.[~~18~~ 19]

There are usually many ways of building barriers to accomplish a certain function. Roofs and umbrellas are substitute barriers against the rain. To stop a pollutant emitted at location $\underline{s}_1$ from reaching a person at $\underline{s}_2$, ~~one~~ we can place a barrier at the source ~~(e.g.~~ smoke control), ~~or~~ at the recipient ~~(e.g.~~ gas mask), or at an intermediate point ~~(e.g.~~ insulated house with filter).

~~One~~ We also may have options to erect a barrier or to take some other action ~~which~~ that obviates the need for the barrier. Two groups ~~which are~~ mutually hostile can migrate away from each other, or they can stay put and erect barriers to reduce contacts ("separation" vs. "segregation").[~~19~~ 20] Or, instead of soundproofing to insure privacy, ~~one can mask~~ sounds can be masked by creating artificial noise. This has been used in connection with church confessionals and physician's examining rooms (not to mention gangland "rubouts").[~~20~~ 21]

~~Let us~~ We now turn briefly from the prevention to the promotion of motion, ~~- that is,~~ i.e., to transportation. Transportation is defined broadly to include any deliberate effort to change location. It therefore includes communication (~~which is~~ the transportation of letters, electromagnetic waves, and other resource types designed especially to convey information), and, for the most part, public utilities (~~which are~~ largely concerned with the movement of water, gas, electricity, and sewage.)

A transportation system may be classified into: the channel, ~~the~~ transmitter, ~~the~~ receiver, ~~the~~ power source, ~~the~~ vehicle, and ~~the~~ cargo. Not all ~~of~~ these components are present in all systems. In automotive transportation, the channel is the road, the power source is internal combustion, the

Multiple-purpose barriers are not uncommon. Thus the Great Wall of China was built to keep the nomads out, but also, and perhaps primarily, to keep the Chinese in.[~~21~~ 22] transmitter and receiver are parking facilities. In radio, ~~one~~ we ~~has~~ have the transmitting station and the radio receiver, with transportation of electromagnetic waves; there is no vehicle or channel in this case. In the sewer system, the channel is the network of sewer pipes, the transmitters are the various toilet facilities, etc., the receiver may be a treatment plant, the power source is usually gravity, the cargo is sewage, and there is no vehicle.

Transportation construction refers to the building of channels, transmitters, and receivers. ~~As already mentioned,~~ it may be thought of as barrier removal or circumvention. Consider the road system. When completed, it establishes a more or less unobstructed surface connecting any two ~~sets~~ sites. (The internal "road" system of buildings — the corridors, stairs, and elevators — may be thought of as a fine-~~structural~~ structured extension of the road system proper; together they connect any two "rooms" in the economy.)

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While road-building reduces barriers to travel along its length, it tends to create new barriers transversely. For example, the building of a bridge creates a barrier to ships too tall to clear it (and thus establishes a lower head of

23 ~~22~~ navigation).~~[22]~~ [23] When roads intersect in a grid system, cross-traffic creates very considerable interference in the form of

24 ~~23~~ slowdowns and extra fuel consumption.~~[23]~~ [24] This sometimes makes it advisable to invest extra resources to reduce ~~this~~ the interference — ~~say~~ e.g., by overpasses, clover-leaf intersections, or

traffic lights. The tradition that the poor lie "on the other side of the tracks" indicates that a transport artery may function as a social barrier.

~~In some cases the transverse interference may be considered an additional benefit rather than a nuisance. The Great Wall of China was built primarily as a barrier against the nomads, but also functioned as a transport artery along its length.~~

FOOTNOTES - CHAPTER 4

F

349 [1]The decline of international violence is more dubious. See L. F. Richardson, Statistics of Deadly Quarrels, ed. by Q. Wright and C. C. Lienau~~, eds.~~ (Boxwood Press, Pittsburgh, 1960).

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353 2. [2]This concept has become familiar through relativity theory, where the finite speed of light plays the rôle of the finite fire engine speed. ~~e.g.~~ See H. Minkowski, "Space and Time", in The Principle of Relativity (Dover, New York, 1923), p. 84.

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355 3. [3]cf. the discussion of "indivisibility" ~~below,~~ section 4.

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360 4. [4]For other examples see A. D. Biderman, M. Louria, and J. Bacchus, Historical Incidents of Extreme Overcrowding (Bureau of Social Science Research, Washington, D.C., 1963).

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365 5. [5]The family of functions f must satisfy the consistency condition: $f_{t_0 t_2}(r,s) = f_{t_1 t_2}(f_{t_0 t_1}(r,s))$. Death could be represented by letting f take on the "non-existence" value z_o, with $f_{t_0 t_1}(z_o) = z_o$; birth could be represented by also introducing "backward causation": $t_0 > t_1$.

t_0 zero

t_0 both

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367 6. [6]For an interesting attempt to model air circulation and pollution in a actual region (Los Angeles) see F. N. Frenkiel,

"Atmospheric Pollution and Zoning in an Urban Area", Scientific Monthly, 82:194-203, ~~April,~~ 1956. H. Reiquam, "Sulfur: Simulated Long-Range Transport in the Atmosphere", Science, 170: 318-320, (16 October 1970) deals ~~1970, is similar.~~ with northwestern Europe.

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368 # 7. [7]In the theory of Markov processes, which the present model resembles, relations of the form (1) are known as Chapman-Kolmogorov equations. If the family of measures represented by f are all simply-concentrated, ~~then~~ this entire construction reduces, in effect, to the dynamical law system discussed above, and (1) reduces to the consistency condition for dynamical laws.

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369 # 8. [8]Remember also that "mass" as we are using the term need not coincide with physical mass. If it does not, (2) is not the same as the physicist's "conservation of mass".

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370 # 9. [9]The well-definedness of (3), and the fact that λ is a measure for each t_1 follows from our assumption that f is a finite conditional measure. For the validity of (4) ~~one~~ we also need~~s~~ to assume that ν is abcont, as of course it would be in this model.

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372 # 10. [10]The classic reference is Activity Analysis of Production and Allocation, T. C. Koopmans, ed~~itor~~ (Wiley, New York, 1951).

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(377) 11. [11] Actually $S \times B = S \times R \times T$ is the order of the components of the cartesian product in (1). No confusion should arise if we permute them to the usual alphabetical order.

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(383) 12. [12] This is the case if all the mass embodied in the set $S \times \Omega_r$ comes from a system of activities in the activity analysis framework of this section. Another more complex possibility is that there are several superimposed systems in operation, e.g., ~~say,~~ one of the activity analysis form, one of the diffusion process form discussed above, and perhaps others. In this case the measure μ will be the sum of the mass distributions involved in the several systems.

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(383) 13. [13] Again, if the activity analysis system is not the only one in operation, μ will relate to the sum of the production measures from the various systems, not to μ_1 alone; similarly for μ_2.

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(393) 14. [14] For further analysis of these and other properties of "production sets", in the context of n-space, see G. Debreu, Theory of Value (Wiley, New York, 1959), pp. 39–42.

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(396) 15. [15] There is ~~an extensive~~ much literature on the effects of extensive scale changes, both in engineering and biology. In the former it goes under the titles "dimensional analysis" or "theory of models". Both aspects are treated in D'Arcy

Thompson's classic work, On Growth and Form (Cambridge University Press, 1917).

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16. For the argument that "non-constant returns to scale" results from "indivisibilities" see, e.g., F. H. Knight, Risk, Uncertainty and Profit (Houghton-Mifflin, Boston, 1921), ~~pages~~ pp. 98, 177; A. P. Lerner, The Economics of Control (Macmillan, New York, 1946), ~~pages~~ pp. 68–69, 143; T. C. Koopmans, Three Essays on the State of Economic Science (McGraw-Hill, New York, 1957), ~~pages~~ pp. 150–154. For criticisms and further discussion see P. A. Samuelson, Foundations of Economic Analysis (Harvard University Press, Cambridge, 1947), ~~pages~~ pp. 84–85; E. H. Chamberlin, The Theory of Monopolistic Competition, 7th ed. (Harvard University Press, Cambridge, ~~7th ed.,~~ 1956), Appendix B; H. Leibenstein, "The Proportionality Controversy and the Theory of Production," Quarterly Journal of Economics, 69:619–625, (November, 1955); D. Schwartzman, "The Methodology of the Theory of Returns to Scale", Oxford Economic Papers ~~(new series)~~, NS 10: 98–105, (February, 1958).

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17. quotation from Stephen Leacock.

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19. Similarly, for the co-evolution of the safe and safe-cracking techniques see E. H. Sutherland and D. R. Cressey, Principles of Criminology, 7th ed. (Lippincott, Philadelphia, ~~7th edition,~~ 1966), p. 275.

18. "Man whilst operating can only apply or withdraw natural bodies; nature internally performs the rest." Novum Organum, Book I, Aphorism 4, in Francis Bacon, Advancement of Learning and Novum [illegible] Book Co., New York, 1900).

406 20. ~~19~~ ~~14.~~ cf. *Genesis* 13: 6–11.

406 21. ~~20~~ ~~30.~~ *New York Times*, May 10, 1964, ~~page~~ p. 40.

407 23. ~~22~~ ~~17.~~ A. E. Smailes, *The Geography of Towns*, 5th ed. (Hutchinson Univ~~ersity~~. Libr~~ary~~., London, ~~5th edition,~~ 1966), pp. ~~pages~~ 47, 54.

407 24. ~~23~~ ~~18.~~ D. M. Winch, *The Economics of Highway Planning* (Univ~~ersity of~~. Toronto Press, Toronto, 1963), pp. ~~pages~~ 67–68.

406 22. ~~21~~ O. Lattimore, *Inner Asian Frontiers of China*, 2nd ed. (American Geogr~~aphical~~. Soc~~iety~~., New York, ~~2nd edition,~~ 1951), p. ~~page~~ 240.