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1. INTRODUCTION

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1.1. Aims, Achievements, and Shortcomings

It is the thesis of this book that measure theory is the natural language for spatial economics, and, indeed, for all ~~of~~ social science.

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As I carried forward the work started in ~~In 1967 the Johns Hopkins Press accepted~~ my dissertation, Essays in Spatial Economics, ~~for publication.~~[1] ~~I set about to make certain minor revisions.~~ It gradually ~~dawned on me~~ became apparent that most of the discussion could be made deeper and more general if it were reformulated in measure-theoretic terms, rather than in the elementary algebraic and geometric language ~~that I - in~~ commonly ~~with~~ used in all standard ~~trantisis~~ treatises on location theory ~~- had been using~~.

no ¶

(At this time I had only the most limited acquaintance with measure theory. (It was mentioned once or twice in my dissertation, but only peripherally.) I resolved to ~~alter my original plan, and fundamentally~~ re-structure ~~the~~ my planned book along these lines, after learning enough of the mathematics of measure theory to ~~carry out this task~~ do so. It eventually became clear that even this fairly ambitious goal did not go far enough. To deal with the problems that arose in location theory ~~I actually~~ it was necessary had to go beyond the existing corpus of mathematical knowledge. As has happened so often in the past - especially with physics - the demands of an applied field stimulated new discoveries in pure mathematics. ~~I believe that~~ this book, taken as a whole, may fairly be said to found a new branch of mathematics - one ~~which~~ that might be called the measure theory of optimization.[2] And not only to found it, but to bring it to a fairly high state of development.

At the same time, ~~I found that~~ the applications ~~were~~ appeared to be broadening: First within location theory, where old results (including my own) were generalized and put on a more rigorous basis, and where new problems were formulated and solved. Theory and applications developed hand-in-hand, each stimulating the other. ~~It gradually dawned on me~~ I came to realize that the framework I had constructed for dealing with locational problems was in a way universal, and could in principle handle all problems of social science.

~~So~~ Thus - one starts with a limited problem and ends up with a ~~vision of the~~ world view. This broadest of extensions is not developed in any detail in the present work, but only sketched (mainly in Chapters 2 and 4). Nonetheless, ~~I would guess,~~ it might be predicted on the basis of these results, that much of the social science of the future will be written in the language of measure theory. but Not all: here the analogy with probability theory is instructive. Since the work of Kolmogorov in 1933, probability theory has come to be viewed as a branch of measure theory. This does not mean that it is worth while ~~taking the trouble to~~ translating every probability problem into measure-theoretic language. But the possibility of such a translation helps one to think clearly and to generalize, and makes available a powerful body of mathematical results ~~should the~~ if they are needed ~~arise for them~~. Similarly, I do not mean that social science will be strait-jacketed into a limited number of categories, or that it will lose its richness of detail or become (further) "de-humanized." Rather, the fact that its statements can be translated into the framework of this book makes for a certain unity of vision ~~which is~~ lacking at present.

~~Let me now turn to a~~ The more detailed discussion that follows will ~~I shall~~ take up the nature of measure theory, ~~the nature~~ of location theory, and ~~the nature~~ of the interpretation of the latter in terms of the former, the problems that arise, and what this book accomplishes and does not accomplish.

Borel Lebesgue

Measure theory will be presented analytically in Chapter ~~1~~ 2. ~~Here~~ however, ~~I take up~~ the subject is treated historically. ~~It~~ This theory originated around 1900 in the work of Bor~~d~~el and Lebes~~q~~gue, and was first directed toward two related problems: to extend the notion of length from intervals to rather general subsets of real numbers in a reasonable way, and to extend the notion of integral to a rather general class of real functions. This program was carried out success-

1.2.2

fully in the first decade of this century, mainly by Lebes~~q~~gue, and the new theory found a number of ~~a~~unexpected applications. The developments ~~which~~ that are really of interest to us occurred, however, in the next decade. First, the new theory was connected with an old idea due to Stieltjes, who in 1894 defined the concept of integration with respect to an arbitrary distribution of mass on the real line.[3] From this more general point of view, the integral of Lebes~~q~~gue is merely the special case in which mass is uniformly distributed over the line. Second, the work of

Carathéodory and Fréchet generalized the concepts of measure and integral from the real line (and its easy extension to n-space) to abstract spaces.

This generality is the key to the applications of measure theory in this book. For the points of the measure space could be, ~~for example~~ e.g., points of (physical) space, or space-time, ~~or~~ personality types, ~~or~~ resource-types in general, ~~or~~ technical processes, ~~or~~ time-paths of development, etc., or even more complicated ~~entities~~ entities built up from these. The measures themselves could represent physical mass, ~~(following Stieltjes),~~ or numbers of events, ~~or~~ dollar values, ~~or~~ man-hours, ~~or~~ volumes, ~~or~~ durations, or degrees of belief, etc. In Chapter 2 it will be shown in detail that most statistical data may be construed directly as measures, and that, by suitable construction, this representation may be extended (in principle) to literally all facts of relevance to social science.

I ~~shall~~ skip over the further historical development of measure theory and conclude with a discussion of the work of Kolmogorov.[4] ~~As noted above,~~ This consisted~~, first of all,~~ of in interpreting the probability calculus as a branch of measure theory. That is, there is a way of translating the vocabulary of probability theory into measure-theoretic terms so that true propositions in the former discipline ~~get~~ are translated into true propositions in the latter. (~~Here are~~ Some examples of such a translation are: "event" becomes "measurable set"; "random variable" becomes "measurable function"; "expectation" becomes "integral"). From this perspective Kolmogorov was able to place probability theory on a rigorous axiomatic basis, and to clarify and develop certain portions ~~of the theory~~ (most notably the foundations of stochastic processes).

Some intriguing issues arise in comparing Kolmogorov's work with the ~~present~~ book. Let me ~~begin by noting~~ note some contrasts. First ~~of all~~, the probability calculus had already a large and impressive body of doctrine; by contrast social science is in a ~~much~~ more primitive state. Secondly, the probability calculus was a branch of pure mathematics (though its foundations were somewhat vague); Kolmogorov's work amounted to an intra-mathematical reduction of one branch to another. By contrast, the present work has the task of representing real world situations in measure-theoretic terms. This requires a much longer and more open-ended discussion. Thirdly, the path ~~lending~~ leading from probability calculus to measure theory is rather short and direct. By contrast, the path from social science to measure theory is a tortuous ~~one~~, involving the construction of a complex conceptual apparatus and considerable development of measure theory itself. For all these reasons Kolmogorov's work could be accomplished ~~in a very short book~~ quite briefly, while the present work had to be ~~quite long~~ much longer.

The most intriguing point of comparison, however, is that both books attempt to reduce their respective subjects to the same mathematical discipline, measure theory. ~~Is~~ this is not a coincidence. ~~No, it is not.~~ To unearth the connection one has to examine the historical roots of probability theory. For, although it is now a self-contained mathematical theory, its basic concepts were designed for the analysis of certain real-world situations, so that it was, and still is, very much an applied theory. Specifically, the work of Pascal and Fermat was directed toward the analysis of gambling games, ~~and~~ from this start the applications were gradually broadened to include insurance, errors of observation, and certain parts of physics, genetics, survey sampling, etc. ~~, etc.~~

Fermat

9.3

To what, then, do probability statements refer? The classical answer is that probability is the number of favorable ~~equi-~~ equally likely ~~eliminating~~ elementary events divided by the total number of such events. This makes probability an additive set function, ~~-~~ in fact, a measure. This additivity property was then carried over to various generalizations: to unequal elementary probabilities, to continuous distributions, etc. Indeed, it is additivity, together with the multiplication rule for combining "independent" events, ~~which~~ that enabled one to speak of a "calculus" of probabilities.

~~To continue the story,~~ this rather crude, almost circular, definition of probability ~~was felt~~ seemed to need clarification. Many different interpretations have been offered, and the question of which ~~of these~~ is "correct" remains a matter of lively controversy to the present day. The major dichotomy is between those who take probability statements to represent "degrees of belief" and those who take them to represent "relative frequencies". A careful discussion would further distinguish several schools within each of these camps. (~~For example~~ e.g., "relative frequency" can refer to a series of observations already made (or which will be made, say, in the next hundred years) or to the limit - if it exists - resulting from the indefinite repetition of an experiment); "degree of belief" can refer to what one *does* believe or to what one should *rationally* believe on the evidence available.)

~~This is not the place to review~~ the relative merits of these approaches will not be reviewed here. Let me, however, ~~briefly~~ indicate their relation to the measure-theoretic translation of probability. Consider first the "relative frequency" interpretation (the "finite" version, not the "limiting" version). ~~Now~~ "relative frequencies" are a special case of the "physical measures" ~~which~~ that are the subject matter of this book. (One goes to the general case in two steps. First, it is usually more convenient to use absolute rather than relative frequencies; second, one often deals with measures in which there are no "natural units", hence no frequencies per se - e.g., the spatial distribution of water: one is "measuring" rather than "counting"). ~~This approach makes it easy possible to retain the classical probability calculus, and its measure-theoretic translation.~~

On the other hand, Jeffreys, de Finetti, and Savage have argued convincingly that for applications of probability some version of the "degree of belief" interpretation must take primacy.[5] Yet, despite much axiomatic work, one is not quite convinced that the probability calculus as it stands today is the proper vehicle for representing degrees of belief. The representability of physical measures is much more firmly established.

~~Finally, let me mention that~~ the framework of ~~the present~~ this book is broad enough to take account simultaneously of both physical measures (including relative frequencies) and degrees of belief. A ~~very~~ brief sketch of how to do this is presented in ~~Chapter 2,~~ section 2.8.

I turn now to the nature of the real-world interpretation; the "coordinating definitions" of ~~between~~ measure-theoretic categories ~~and the real world~~. ~~I am not~~ My concern~~ed~~ here is not with details, which will be amply covered in chapter 2 and throughout the book, but with broad ideas and motivations.

The basic observation is that ~~one can~~ can be describe d the world by stating how much matter is embodied in what forms at what places at what dates. Or, more generally, how much matter is undergoing such-and-such transformations at what places at what dates.

~~Now~~ The language of measure theory is ready-made for descriptions of this sort: Quantities of matter are represented by the values of measures, while the underlying universe set whose subsets are given these values is built up from the fundamental categories of Resources, Space, and Time, describing forms, places, and dates, respectively. (~~How one~~ Methods of deal~~s~~ing with transformations and other more complex descriptions ~~is~~ are spelled out in chapter 2).

Measurement need not be in terms of physical mass, but may be in terms of numbers of units, volumes, money values, etc., as befits the particular entities under discussion.

START
9.2.4

But as soon as one realizes ~~that~~ it is possible to describe the world in measure-theoretic terms, it becomes clear that one can also analyze the world in these terms. Laws of development can be formulated in measure-theoretic language, problems can be posed and solved in it, ~~and so on.~~ etc.

The question remains, If ~~granted that one can use~~ measure-theoretic language can be used in this way, is it worthwhile to do so? Why should it be more useful to translate into measure-theory than to translate into, say, Esperanto? The only answer is to make the translation and see what ensues ~~one gets~~. This book shows that one can obtain a massive and rather imposing body of results, and it seems clear that these ~~results obtained here~~ are just a small part of the results that await further research along these lines.

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We all have the experience of working fruitlessly on a problem for a time, ~~and then having~~ only to see the solution ~~come~~ clear when we re-formulate the problem in the right way. Call this the "natural" formulation for the problem in question. Now think, not of one problem, but of a whole range of problems constituting some discipline. And think, not of a single formulation, but of a range of interrelated formulations constituting a consistent point of view, a method, a body of doctrine, a theory, or whatever you wish to call it. If it often happens that problems from some discipline yield when formulated in terms of a certain theory, one may speak of that theory as the "natural language" for that discipline.

~~I believe that~~ Measure theory is the natural language for spatial economics in this sense. Let me ~~briefly~~ examine the status of this latter discipline. If one compares the standard tr~~i~~eatises on location of the 1930's with those of the present,[6] one's general impression is that not much progress has been made — at least relative to the rest of economic theory, much of which has been transformed almost beyond recognition over this period. Why should this be? Certainly not from lack of interest. Indeed, regional and urban economics - which are the applied parts of the discipline - have grown enormously ~~in this period~~. Nor is it clear that location theory is so much more difficult than other branches of economics. ~~Indeed,~~ "space" is really a very simple category. Transportation is much simpler conceptually than most manufacturing processes. A location can be characterized by three (or two) numbers, while the state of a person or a commodity requires an indefinitely long description. So, with such a simple subject matter, why is the field so backward?[7]

~~As indicated above, my~~ The answer is that location theorists have up to now simply not found the proper language in which to formulate their problems~~, and that~~ measure theory is that ~~the proper~~ language. This entire book is an argument for ~~the second half of~~ this statement. ~~As for the first half,~~ let me ~~briefly~~ indicate the kinds of snarls ~~one runs into~~ that are encountered if one does not use measure theory.

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Consider the theory of Thünen systems (Chapter 8 ~~below~~). Von Thünen's work dates from 1826 and contains the oldest formal model in location theory. The problem is to determine the spatial distribution of agricultural land uses, given that all deliveries are to be made to a single isolated city (thought of as a geometrical point). There are several basic conceptual difficulties. First, what is a "land use"? An indefinitely large variety of commodities exist ~~which~~ that may enter as inputs or outputs in a land use (e.g., the different possible quantities of corn or fertilizer); the proportions in which these commodities enter may vary continuously; finally, there may be indefinite variations in the time-distribution of these inputs and outputs (sequencing, seasonal cycles, etc.). Clearly, "land use" is no simple concept. The complications become even worse with the modern observation that the Thünen analysis is relevant to the spatial distribution of <u>urban</u> land uses, so that it becomes a theory of the internal structure of the city. For this application the concept of "land use" must be extended to include various kinds of residential processes, manufacturing, trade, services, office activities, etc. Furthermore, it is desirable to distinguish various time-patterns within each of these uses, as well as the succession of different uses on the same site.

g 2-5

Existing studies deal with these possibilities by drastic simplification, either by assuming there are just a finite number of possible land uses, or by assuming they may be described by the variation of a few parameters, (e.g., "intensity"). By contrast, the measure-theoretic approach easily accommodates all the complications mentioned above.

A second difficulty involves the concept of "spatial distribution" of land uses. There is an infinite variety of possible distributions even for the case of just two land uses: Corn and oats may be mixed in proportions that vary with location, or they may be segregated (in which case the region assigned to each must be specified)~~, etc~~. For the general set of land uses as discussed above, the possibilities are of course much richer. Existing theories usually cope with this difficulty by restricting a priori the set of distributions - e.g., assuming that land uses are segregated into geometrically simple regions. The measure-theoretic approach, ~~on the other hand~~ however, accommodates the general case.

A third difficulty involves the allocation of "continuous" space to "discrete" decision-makers, e.g., farmers. The solution offered in ~~Chapter 8, section 7,~~ 8.7 seems conceptually superior to that of ~~predicting~~ preceding writers,[8] though the assumptions concerning preferences may well be criticized as unrealistic.

The "snarls" in traditional approaches ~~which we have mentioned so far all~~ involve inadequacies of descriptive power. More serious are their inadequacies of analytical power. Consider the normative analysis of Thünen systems, which is the main focus of Chapter 8. The question of whether, or in what sense, the Thünen arrangement of land uses is optimal, is unresolved in the literature.[9] It turns out that the Thünen solution <u>is</u> optimal for a certain simple linear-programming transportation problem! This is Not your ordinary transportation problem, however, but a measure-theoretic generalization of it. (See Chapter 7.) Indeed, applying the ordinary transportation problem in this way requires a most unnatural "discretization" of space, as well as a restriction to a mere finite number of land uses; thus the measure-theoretic generalization is needed to deal adequately with the Thünen problem.

The Thünen model arises from the interplay of a single "discrete" city and a "continuous" hinterland. More generally, this interplay of "discrete" and "continuous" pervades location theory: A "discrete" plant pollutes a "continuous" environment; a "discrete" policeman patrols a "continuous" neighborhood; a "continuously"-dispersed rural population condenses to a "discrete" city.

This interplay is another source for the descriptive and analytical superiority of measure-theoretic over traditional approaches. Measure-theoretic description allows for "discrete", "continuous" and "mixed" distributions over Space (e.g., urban-rural population), over Time (e.g., "lumpy" and "continuous" production), and over Resources (e.g., positive amounts of some resource, together with a "continuous" distribution over others), as well as over more complex spaces. As for analytic power, note first that many problems can scarcely be formulated without measure-theoretic language; e.g., is it optimal for a certain industry to be spatially distributed into "discrete" plants, or "continuously", or with a "mixture" of these, etc.?[10] Most of the models of Chapter 9 involve this "discrete" - "continuous" interplay.[11]

These examples should give some inkling of the sense in which measure theory is the natural language for spatial economics. ~~As~~

~~noted above,~~ it lends the subject generality, unity, and mathematical power; above all, it is an aid to clear thinking ~~helps one to think clearly~~.

~~Now~~ spatial economics is not just another branch of learning.[12] That Space (and Time) are fundamental categories in any attempt to understand the world is an old idea, going back at least to the Greek atomists. If ~~one tries~~ to put specify ~~one's finger on~~ the somewhat elusive reasons for this notion, one arrives at the following intuitive arguments: First, their <u>universality</u>: Everything has a location and a date. Second, their <u>universal requiredness</u>: Every activity needs room in which to operate, and a certain duration in which to be consummated. (Thus the classical economists spoke of "land" and the "period of production".) Third, their connection with <u>causation</u>: Things must be in contact to interact. (More generally, they may interact indirectly, e.g., by sending signals through the communications system; more generally still, the intensity of interaction depends on distance and relative position).

§2-6

This ~~The present~~ book continues this "spatio-temporal" tradition, the detailed constructions occurring mainly in chapters 2 and 4. To assure that it does not become just another abortive attempt at system-building, to be forgotten in its turn, I have concentrated on turning out a body of results too large and impressive to be ignored. This again explains the size of the book.

Let me now try to summarize what this book does and does not accomplish.

The accomplishments have already been discussed. First, the book takes a quantum jump over existing location theory, in generalizing existing results, in putting them on a more rigorous basis, and in striking out in new directions. I have not attempted to write a comprehensive treatise on location theory, but only to cover ~~those~~ the parts where I have succeeded in making substantial progress via measure-theoretic formulation. ~~Those parts,~~ These however, do constitute the bulk of the "mainstream" tradition of von Thünen, Launhardt, Weber, Palander, Hoover, Lösch, Ponsard, Isard, Alonso, and Beckmann. ~~and Richardson.~~[13] Second, the book develops the mathematics of measure theory itself. The development is ultimately directed toward applications, but ~~a good deal~~ much of it should be of considerable interest even to the "pure" mathematician. Third, the book develops a framework of ideas ~~which~~ that seems to be adequate for all ~~of~~ social science, not just location theory.

~~I now come to the~~ things ~~which~~ this book does not accomplish. It is important to take full note of these, to gain perspective and to answer the question, "where do we go from here?" I ~~will~~ divide these "non-accomplishments" into four groups: theoretical, foundational, empirical, and practical.

Despite its bulk, the book discusses only a ~~minute~~ fraction of the relevant theoretical models. I have concentrated on optimization models and have slighted, though not ignored, equilibrium models.[14] The latter ~~models~~ are partial, not general (in the sense in which economists use these terms; each discipline has its own particular notion of what is "general"). Almost all the models are deterministic (though some could be interpreted as applying to expected values).

The sense in which these models are or are not dynamic requires some discussion. The overall framework of the book has Time as one of its basic categories, and allows one to describe change and development without trouble. But a dynamic model in addition puts restrictions on the kinds of changes which that can occur. Now there are several ways of specifying these restrictions. One is to state "laws of development", say in the form of differential equations. In this sense very few of the models are dynamic. Another approach is to specify a set of feasible "activities", each "activity" being defined by a complete input-output structure over Time. Here change is determined implicitly by the choice of activity. This latter approach is the one used in Chapter 8, and (to some extent) in Chapter 9. Still another approach is to consider Time as simply a fourth spatial dimension. This is the approach used in the real-estate model of Chapter 6. (These and other approaches are considered in Chapter 4).

One would like to introduce non-linear features into several of the models. ~~For example~~ e.g., preferences are usually expressed by integrals and are thereby linear in the corresponding measures and integrands. But even in the realm of the linear there is plenty of room for generalization. One expects that linear programs in general can be put into measure-theoretic form along the lines of Chapter 7, which is restricted to transportation and transhipment problems.

S2-7

¶ ~~So much for the theoretical gaps.~~ The "foundational gap" is the one between the conceptual framework of this book and the world of experience. The claim is that one can describe the world in terms of measures; but the description is a highly formalized one, several steps removed from everyday perception. Specifying the values of a measure involves a process of measurement ~~which~~ that needs elucidation. How does one recognize the same location at different times? the same physical object persisting through time? the same resource-type at different times or places? What is a resource-type anyway? What is the significance of countable additivity? of measurability?

All of these questions deserve extended discussion. ~~I have in fact~~ Some are discussed ~~some of them~~ in Chapter 2, but only cursorily. I have concentrated instead on setting up the structure, not worrying overmuch about the foundations. One must first attain a body of results, and only then inquire as to what they mean. ~~This is the usual sequence in science.~~

As for the "empirical-practical gaps", remember that this book is intended as a work in theory: It is not concerned directly with particular real-world situations. On the other hand, it is important to have a large number of links between the theory and the real world, both to clarify the concepts and to indicate directions of application. Accordingly, ~~there are~~ literally scores of illustrations of theoretical models are scattered throughout the text. These illustrations should be understood as hypotheses that the suggested interpretations are reasonable approximations to the real-world situation. For example, in Chapter 8 I suggest that the observed tendency for population density to be higher along the fringes of continents than in their interiors is explainable along von Thünen lines. In effect, this draws attention to a small number of facts – (e.g. water vs. land transport cost differentials) – which with some simplifying assumptions yield the general configuration observed; other relevant facts, such as climatic differentials, are ignored, and would be incorporated in a more complete theory.

~~What is~~ lacking here is a "spatial econometrics", a body of doctrine to guide statistical inference for the models of location theory. In addition, of course, in any particular application a detailed factual investigation would be needed to assess the "goodness of fit" of the models, and not merely the "casual empiricism" ~~which is~~ appropriate for this book.

Finally, there are the "practical gaps", the absence in most cases of feasible computational schemes for obtaining solutions to the various models. It is one thing to show that a certain equilibrium exists, or that a certain problem has an optimal solution, and quite another to calculate these solutions with a reasonably small amount of computational effort. ~~Actually,~~ I have indicated in some cases how this might be done: usually successive approximation schemes via price adjustments. But these ~~I~~ have not been carried ~~these~~ out in detail.[15]

In this connection, there is the "practical-minded" man's objection to the whole enterprise of this book. The argument might go as follows: "You have this (allegedly) wonderful new theory. But how does it help me meet a payroll? Specifically, a measure is, in general, a very complicated object. To reduce information handling costs, you have to aggregate it to a discrete distribution. But if you do this, doesn't your theory collapse to something we all know already – ordinary linear programming, for instance?"

I will answer this objection in three stages. First, for practical work one must certainly approximate by a family of objects ~~which~~ that can be indexed by a small number of parameters. But there are many ways to do this. Population distribution over a region may be represented by partitioning into a subregions and giving the population of each (discrete aggregation). It might be more convenient, however, to approximate the distribution of a city population by a circular normal or exponential distribution, and the population of a region by a sum of such measures. How best to approximate is itself a genuine problem, which can be solved only by taking the "raw" measure in all its complexity as a starting point. Second, for many of the problems in this book, though they involve unrestricted measures, it can be shown that they may be indexed by a small number of parameters. The Weber problem on

§2-8 the plane (Section 9.4) has two parameters; the market-region problem with n plants (Section 9.5) has n or n-1 parameters (viz, prices); the

8 Löschian one-industry problem with fixed deliveries (Section 9.6) has two parameters (scale and spacing); the interplant and inter-industry problems of Section 9.3 can be reduced to combinatorial problems. Furthermore, several of the problems have solutions ~~which~~ that have very simple characterizations in terms of the given data; these include

the allotment-assignment problem (Section 8.5), and some of the police-criminal-victim models of Section 5.8. Thus, all of these models are "computable". What one should ~~do~~ be done with the remaining models, such as the transportation problem of Chapter 7, from the practical point of view remains an open question. It is hardly justified, however, to replace these problems a priori by discrete aggregations (several of the "computable" problems above are in fact special cases of the transportation problem).

The third stage of the rebuttal takes a different tack. Suppose for the sake of argument that none of the models in this book were computable. Would it still have been worth writing? My answer is yes. ~~As discussed above,~~ the theory lends a certain power and clarity to thought, which must issue indirectly in a firmer grasp of practical problems. And the overall view of the world ~~which~~ it contains has a certain grandeur.

1.2. Synopsis of Chapters

Chapter 2. This chapter has two objectives: to serve as an exposition and reference to the measure theory used ~~in this book~~ here, and to lay out the basic conceptual framework of the book. Sections 2.1, 2.4, and 2.6 are devoted to measure theory. Measures are defined in section 2.1, and section (2.2) then introduces the discussion of how real-world data may be represented as measures. Sections 2.3 and 2.4 are preliminary to section 2.5, which sets out the general theory of the measure-theoretic representation of the real world. Section 2.3 introduces the fundamental categories of Space, Time, and Resources, while section 2.4 introduces the measure-theoretic concepts and theorems needed to understand section 2.5. The major exposition of measure theory occurs in section 2.6. Section 2.7 introduces activities and related concepts; these are essentially aids to picking out interesting patterns from the all-embracing flux of section 2.5. Finally, section 2.8 notes several real-world phenomena whose representation requires multi-layer measures, ~~that is~~ i.e., measures over a space whose points are themselves measures.

How does the exposition of measure theory here compare with that in standard treatises? I have stressed measures over <u>abstract</u> spaces rather than over the real line or <u>n</u>-space. This is not just idle generality, but is needed because many of the spaces dealt with are rather complicated entities, not easily reducible to the real numbers; these include the space of Resources, of Histories, of Activities, multi-layer spaces, etc.

Most of the material covered in standard treatises appears here, and conversely most of the mathematical material in this chapter appears in standard treatises. But the overlap is not complete in either direction. For example, I have omitted any discussion of measures on groups, differential measure theory, vector measures, Hausdorff measures, complete measures, outer measures, Daniell integrals, and Lebesgue decompositions - simply because these concepts are not used in the remainder of the book. Furthermore, to save space I have omitted all proofs of standard theorems (except occasionally when these are very short). Hence the reader who wants a deep understanding of the subject should by all means consult the standard treatises even for the topics ~~which are~~ covered.

~~In the opposite direction,~~ some results ~~which~~ that appear to be new are covered in section 2.6, in particular in the subsection "Abcont and Product Measures" (where full proofs are provided), and also in the subsection "Extension of Set Functions".

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As for the "world-representation" whose exposition culminates in section 2.5, the reader acquainted with the theory of stochastic processes will note the resemblance to the concepts used in that subject. But note also the differences: Measures are not normalized, and may be infinite; more significantly, the "realizations" (here called "histories") do not all have the same time-domain; finally, of course, the interpretations of the concepts used ~~is~~ are completely different from ~~that~~ those common in stochastic processes.[16]

(B) Chapter 3. Mathematically, ~~this~~ is the deepest chapter in the book. The theory generalizes the concept of signed measure. Section 3.1 works with measures in general. Section 3.2 specializes to ~~sigma~~ σ-finite measures, leading to the fundamental concept of pseudomeasure. Algebraic operations and integration with pseudomeasures are defined in this section, while various order relations are defined in section 3.3. There are two basic types of applications of pseudomeasures in this book. First, pseudomeasures may be used to represent "net" values (e.g., net production, net migration) even in ~~those~~ cases where both "gross" measures are infinite, as might occur in models with infinite horizons. The main application of this type is to the transhipment problem, Sections 7.7 ~~through~~ – 7.11 (see also Section 6.9). Much

pseudomeasure

more important is the use of pseudomeasures to represent preferences. Pseudomeasures are used throughout the book in this way, and this application allows the formulation of models ~~which~~ that are both more general and simpler than would otherwise be the case.[17] The use of pseudomeasures also generalizes neatly such apparently unrelated subjects as the Ramsey-Weizsäcker "overtaking" approach and the Bernoulli-von-Neumann-Morgenstern expected utility approach, the former by going from Time to more general spaces, the latter by allowing unbounded utility functions. These topics are discussed in section 3.3.

(a)

Pseudomeasure theory is quite elegant from a purely mathematical point of view, and chapter 3 is just a bare introduction to it. The subject is developed further in sections 6.9 and ~~in~~ 7.11.

(B) Chapter 4. While chapter 2 contains a descriptive framework valid in "all possible worlds", so to speak, chapter 4 is devoted to various approaches to the representation of "attainable worlds", ~~—that is~~ i.e., to the representation of an agent's power. As introduced in section 4.1, power is limited in many different ways: by lack of knowledge, by natural law, by resource availability, by limited authority, by legal constraints, by budget constraints, etc. These are all discussed in chapter 4 (except for budget constraints, discussed in section 6.1). Section 4.2 elucidates the concept of uncontrollability, e.g., the sense in which the past is uncontrollable. Section 4.3 discusses cross-sectional constraints, ~~that is~~ i.e., constraints on the possible distributions over Space and Resources at a moment of Time. Section 4.4 explores a few simple cases of the potentially much richer class of constraints involving several moments of Time; it ends with an example involving pollution, essentially a simple diffusion model. Section 5 builds on the activity concepts of 2.7. It first specializes to the case of "simply-located sedentery" activities, that is, activities located at a single fixed point. The construction which follows is essentially a measure-theoretic generalization of the standard "activity analysis" approach. Sections 6 and 7 examine certain assumptions implicit in this approach. In section 6 the assumption that there are no "neighborhood effects" is explicated and then criticized as to its realism. In section 7 the same is done for the assumption of "constant returns to scale". Three different meanings of this concept are distinguished, stemming from three different scale concepts discussed in 2.7. Section 8 briefly discusses the Knightian contention that "indivisibility" is responsible for "non-constant returns to scale." Six different meanings of "indivisibility" are distinguished, none of which support Knight's contention. (The discussion in sections 7 and 8 - which is by no means definitive - is an example of how explicit consideration of the spatial variable can clarify certain unsettled issues in economic theory). Finally, section 9 is an informal discussion of the modes by which one controls the location of things (and thereby their interaction): bringing things together, keeping them apart, maintaining relative positions, etc., by means of barriers, walls, bindings, transportation construction, etc.

Chapter 4 is the most unfinished ~~of all the chapters.~~ The discussion is at a rudimentary level, and there is no overall point of view uniting the sections as ~~there is~~ in most other chapters.

Chapter 5. ~~This~~ chapter is devoted mainly to the study of one very general and pervasive type of optimization problem, an example of which is: Given alternative projects, returns from each depending on the amounts of effort invested, with constraints on the total effort available, and on the amount that may be invested in each project, allocate effort among projects to maximize overall return. Section 1 discusses various interpretations: Effort may be money, time, resources, personnel, etc. The formal problem is in one sense a special case of the Neyman-Pearson problem, special in that it has just one allocative constraint; but it is extremely general in all other respects. The problem is formally set up in section 2, and analyzed in sections 3 ~~through~~ 7. In particular, in section 4 a "shadow-price" condition associated with optimal solutions is derived. Finally, in section 8 these results are applied to the problem of explaining the spatial distribution of crime. This is determined by a game between Criminals and Victims on the one hand, and between Criminals and Police on the other. In these games the participants are faced with allocation-of-effort problems. The results predict a positive association between crime rates and population (or wealth) density, and also predict a tendency for crime to "suburbanize" with increased numbers of police.

The allocation-of-effort problem finds further applications in chapters 8 and 9.

Chapter 6. Sections 1 through 3 focus on the budget constraints facing a single agent. Section 1 stresses basic accounting identities in perfectly competitive markets, spread over a measurable space of commodities at various times and places. Section 2 extends the analysis to rental markets, discussing ownership vs. control, bailments and service activities. Section 3 indicates how the measure-theoretic approach extends to encompass imperfections in the commodity and/or capital markets. The discussion is rudimentary throughout.

Section 4 introduces the main topic of this chapter, the real-estate market, which allocates the ownership and control of Space-Time. Discussion of the organization of this market carries over into section 5, which stresses the question of how best to represent the preferences of the participants in the market, balancing simplicity against realism. Assuming these simplifications, section 6 proves the existence of equilibrium in the real-estate market and explores certain of its properties. Section 7 indicates how joint control of real-estate among several agents might be represented, in contrast to the exclusive control approach of section 6. Section 8 is a critique of a leading alternative theory of the real-estate market, that of Alonso. Finally, section 9 generalizes the analysis of section 6 in a direction which might be relevant in unbounded Space or Time, where the number of agents might be infinite, rents might be infinite, preferences no longer representable by real numbers, etc. These complications are all handled by introducing pseudomeasures, and the analysis closely parallels that of section 6.6. An appendix to section 6.9 develops certain order properties of the space of pseudomeasures ~~which~~ that grow out of this analysis.

An alternative approach to the real-estate market occurs in ~~Chapter 8,~~ section 8.7. ~~Briefly,~~ this latter model is deeper than the ~~present~~ one in this chapter in that preferences over regions are derived from the uses one is going to run there, rather than being assumed directly. ~~On the other hand,~~ However the Section 8.7 model operates in a Thünen context, while the ~~present model~~ one in Chapter 6 has no such restriction.

Chapter 7. ~~The~~ A chapter 7 develops the measure-theoretic generalization of the transportation (sections 7.1 ~~through~~ – 7.5) and transhipment (sections 7.6 ~~through~~ – 7.11) problems of linear programming. Capacities and requirements are represented as measures, and cost as an integral (in general, a pseudomeasure). Topological concepts are introduced as needed. The section headings are mostly self-explanatory. "Potentials" in section 7.5 refer to the system of "shadow-prices" associated with optimal solutions. Section 7.6 discusses the relation between the two problems, and the realism of their assumptions. (Further connections between the two problems are obtained in the latter part of section 7.7 and in section 7.10). The "skew" problem of section 7.11 is the transhipment problem in terms of net shipments. It is interesting that in one formalization the problem is to find an optimal pseudomeasure.

(All other problems in this book involve finding unknown measures, ~~or~~ point functions, ~~or~~ individual points, or sets).

The transportation problem is the foundation of chapter 8, and both transportation and transhipment appear in various sections of chapter 9.

B

Chapter 8. Chapters 8 and 9 (and chapter 6 to a lesser extent) ~~are the ones which~~ deal with the traditional models of location theory, though ~~of course~~ they go far beyond the standard agenda of topics. ~~This~~ chapter 8 deals with Thünen models, introduced in section 8.1. Stress is placed on the optimization aspects of these models, though equilibrium is covered as well. Section 8.2 introduces "ideal" weights and distances, which are needed because real-world transport costs are not proportional to ton-miles of shipments. The special modifications of these concepts needed in the Thünen context are discussed in section 8.3. Section 8.4 ~~holds~~ takes off from the activity-analysis framework of chapter 4. It indicates how "amounts" of land uses may be measured in "ideal" acres. ("Ideal" area is the third ideal concept used; it allows for "fertility" differentials and land availability not proportional to physical area; "land uses" are simply space-using activities). Also it indicates how resource-time ideal weights translate into land-use ~~weights~~ weights. These formalities allow much simplification in the expressions for transport costs and areal capacity constraints. Interpretations of land uses in terms of time-patterns, multiple stories, and ideal weight assessments are discussed.

(ii) Section 8.5 gives the basic Thünen optimization model, to minimize overall transport costs subject to areal capacity and land-use allotment constraints. Formally, this becomes ~~turns out to be~~ a special case of the measure-theoretic transportation problem. A detailed analysis follows, the special features allowing sharper results than in Chapter 7. Section 8.6 then applies these abstract results to concrete Thünen systems, and also gives several other applications, some far removed from the original spatial context.

Section 8.6 gives the basic Thünen equilibrium model. Each agent must decide which land to acquire and what land uses to run there. ~~on his land.~~ Under fairly weak assumptions concerning preferences, etc., it is shown that any equilibrium will satisfy the Thünen pattern in the overall spatial distribution of land uses and land values. In view of the very complicated arguments needed for rigor, a simplified heuristic approach is also given (taken from my *Essays*).

(ii) §2-12 Section 8.8 discusses many interpretations, predictions, and informal extensions of this very flexible model: the spatial distribution of multiple-story structures, land speculation, people by income, family size, and car ownership; the influence of pollution, crime, and danger; the effects of taxation; and the welfare implications of the model. ~~Finally~~, Section 8.9 discusses a variety of real-world Thünen systems classified in several ways. First, by spatial scale: from workbenches to theaters, farms, villages, city neighborhoods, cities, metropolitan regions, and

(ii)

sub-continental systems, ta the entire world. The latter, coupled with ideal distance distortions, explains the tendency for continents to have relatively empty interiors. Second, dynamically, in particular whether they develop by inward migration (e.g. rural-urban, dual economics) or by outward migration (e.g. sub-urbanization, pioneering). Thünen systems fall into a hierarchial structure, and this yields an explanation of the "central places" arrangement. Some indices for assessing Thünen fits are noted, such as transport nets, Stewart potentials, percentage of non-local flows.

Chapter 9. Section 1 discusses informally a variety of preference patterns in a spatial context, involving pollution, layout problems, attraction and repulsion among persons and groups, density preferences, visits to facilities, etc. Section 2 discusses a number of informal models involving interacting agents having some of these preference patterns. Specifically, agents are assumed to be attracted or repelled from various others, and can ~~set~~ act by themselves moving or by inducing others to move (or to stay put). Another group of models involves attraction to facilities or services for which the agent has a taste, leading to generalizations of the Tiebout model, urgan neighborhood formation, segregation by income, etc. Changes in preference are briefly discussed, such as fatigue, deprivation, extinction and habituation of tastes.

The rest of Chapter 9 is more formal in tone. Section 3 deals with certain inter-plant and inter-industry optimization problems, in which flows between all pairs of plants or industries are given, along with the location of some plants or industries, and the others are to be located to minimize overall transport costs. Under certain conditions there is a tendency ~~in~~ for optimal locations to coincide, a clue to the observed tendency toward agglomeration in the real world. The concept of "orientation" of an industry is clarified, and an explanation is given for the tendency of industries to dichotomize into "resource" and "market" orientation. A different model deals with the "localization" of industries into small regions.

Section 9.4 deals with the classical Weber problem, to choose a plant location minimizing transport costs, given the spatial distribution of shipments to and from the plant. A multiplant generalization is also considered, in which interplant flows are also given, and all plants must be located. Many interpretations are ~~given~~ offered, along with a discussion of subproblems in general. The connection with "nodality" is discussed. Results concerning existence and bounds on solutions are obtained, as well as the use of symmetry and convexity in finding solutions. Finally, the multi-plant Weber problem in a Thünen context is examined, with some surprising results.

Section 9.5 deals with a class of problems of which the following is typical: Given a system of plants, with locations and capacities specified, and given a spatial distribution of demand, find a shipping pattern that satisfies these constraints at minimal transport cost. This is a special case of the transportation problem of Chapter 7, special in that the origin space is countable. Just as with the allotment-assignment problem (Section 8.5), this special feature leads to sharper results. Specifically, optimal flows are associated with a system of ~~shadow~~ Shadow mill prices ~~which~~ that determine market regions, each plant shipping exclusively to its own region. The implications between these prices, various market region concepts, and optimal flows are explored in detail.

9.2-13 In the "service systems" of section 9.6, finally, one is to find ~~both~~ the number, location, and output of plants, as well as and the spatial distribution of outputs, both from the viewpoint of optimization (maximizing total benefits minus transport and production costs), and for various social equilibria. A classification scheme is set up and the "goodness of fit" of various real-world systems to the model is discussed. The case of a single plant with fixed location and output is taken up first; this reduces to an allocation-of-effort problem (Chapter 5). Then the discussion moves on to many plants, again with all locations and outputs fixed; the market region problem (section 9.5) ~~turns out to be~~ is a subproblem of this. The analysis turns on the relations between optimal solutions and the "shadow-price" conditions of the market region and allocation-of-effort subproblems. The next topic is the relation between spatial prices and shipment patterns, in particular the Mills-Lav demonstration that parts of the field may go unserved. Finally, the difficult subject of variable plant locations and numbers is broached. The need (for the first and only time in this book) for introducing extended ordering of pseudomeasures is noted.

The inefficiency of the honeycomb lattice arrangement of plants is demonstrated.[18] This is contrary to many statements in the literature, and indicates the unsettled nature of the subject.

A

1.3. Reader's Guide ~~to this Book~~

This is a difficult book for several reasons: ~~-~~ its length, its novelty, the inherent complexity of its arguments, and the fact that its central mathematical apparatus, measure theory, is at present unfamiliar to most economists and other social scientists. Hence the need for a "reader's guide".

One pervasive problem ~~that I faced~~ in organizing the book was the double task of developing the formal theory on the one hand, and connecting it with real-world concepts on the other. The first task calls for a rigorous self-contained approach; the second ~~calls~~ for an open-ended intuitive approach. I have tried to accomplish both tasks by~~, first of all,~~ segregating the formal and intuitive portions into separate sections; this has ~~I have~~ been only partially successful ~~in this~~. My main device has been to signal ~~the~~ formal discussion by using ~~writing~~ Definition, Theorem, Proof, and ||| at the end of a proof. The text thus blocked out ~~by these signals~~ is meant to be self-contained and to meet full standards of mathematical rigor (in contrast to much contemporary literature that ~~which~~ is studded with pseudo-"proofs" of "theorems").

The only mathematical prerequisite for ~~reading~~ this book is elementary calculus (e.g., real numbers, limits, mean-value theorem, absolute convergence, uniform continuity, Riemann integral), though the reader whose knowledge stops at this point will not have an easy time. All other concepts and theorems are developed as needed.

It should be possible to understand the text by skipping all proofs, reading only the definitions and theorems, ~~and~~ thereby avoiding the most difficult parts of the book. Reading the proofs, of course, is needed for a really deep understanding of the theories developed (as well as for verifying my results).

The diagram below indicates the precedence relations among the chapters, the solid arrow ~~→~~ indicating strong dependence, and the broken arrow ~~---→~~ indicating weak dependence. Getting down to the section level, Chapter 4 is needed for section 8.4, Chapter 5 is needed for sections 8.6 and 9.6, Chapter 7 (transportation half) is needed for sections 8.5 and 9.5, Chapter 7 (transhipment half) is needed for section 9.3, Chapter 8 is needed for section 9.4.

$$\begin{tikzcd}
(2) \arrow[rr] \arrow[d] & & (3) \arrow[rr] \arrow[dl] \arrow[dr] & & (6) \\
(4) \arrow[dr, dashed] & (7) \arrow[d] \arrow[drr] & & (5) \arrow[d] \arrow[dll, dashed] & \\
& (8) \arrow[rr, dashed] & & (9) &
\end{tikzcd}$$

diagram about here

C.C. ~~[illegible]~~ in H2

Chapter 2 is precedent to all other chapters. Perhaps sections 2.1 ~~through~~ – 2.5 ~~of Chapter 2~~ should be read straight through, and section 2.6 used as a general reference on measure theory.

G 2-14

Chapter 3 presents a special problem because of its novelty. ~~As indicated,~~ it is precedent to all other chapters except 2 and 4; but the reader who does not feel at home with pseudomeasures may circumvent this dependence, at least in part: first, many sections in Chapters 5 ~~through~~ – 9 do not use pseudomeasures. Second, the standard integral theorem ~~(Page)~~ (p. 000) allows one in

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some cases to translate a utility function expressed in pseudo-measures into more familiar terms (this works in Chapter 5, for example, but not in most of Chapter 7); third, one may add extra conditions, such as boundedness, to various premises to insure that utilities are expressible as real numbers, thus circumventing Chapter 3 at the cost of some generality. All this still leaves a residue of results ~~which~~ that require a working knowledge of pseudo-measures, which ~~is~~ are really not all that difficult to master. (Note that Chapter 3 has to be read only through standard order; extended order is used only in Section 9.6, ~~at the very end of the book,~~ except for passing references).

~~X. Y indicates section Y of chapter X.~~ Differentiation is ~~indicated by D. All other notations are standard or explained in the text.~~

(add note explaining section numbering system)

Section Y of Chapter X is indicated by X.Y. Formula Z of Section X.Y is indicated by (X.Y.Z). It is referred to as (Z) within Section X.Y, referred to as (Y.Z) elsewhere in Chapter X, and referred to as (X.Y.Z) in other chapters.

FOOTNOTES, Chapter 1

A.M. Faden, Essays in Spatial Economics,

F

1. [1]Ph.D. Dissertation, ~~Department of~~ Economics, Columbia University, 1967.

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2. [2]Certain existing disciplines fall under this rubric, notably parts of the theory of games and parts of mathematical statistics. But ~~the present~~ this work, book ~~-~~ with the exception of Chapter 5, ~~-~~ is independent of these. It is also independent of the one existing direct application of measures in economic theory, the "continuum ~~continuous~~ of traders" model. For comments on the latter, see (Section 6.7.) ~~pages below.~~ ~~(i.e., chapter 6, section 7).~~

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3. [3]T. Hawkins, Lebesgue's Theory of Integration: Its Origins and Development (~~University of~~ Wisconsin Press, Madison, 1970), pp. ~~pages~~ 180–181.

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4. [4]A. N. Kolmogorov, Grundbegriffe der Wahrscheinlichkeits-rechnung (Springer, Berlin, 1933), ~~translated by~~ N. Morrison as Foundations of the Theory of Probability 2nd ed. (Chelsea, New York, ~~2nd edition,~~ 1956).

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5. [5]H. Jeffreys, Theory of Probability 3rd ed. (Clarendon Press, Oxford, ~~3rd edition,~~ 1961), especially Chapter 7; Studies in Subjective Probability, ~~edited by~~ H. E. Kyburg, Jr., and H. E. Smokler, eds. (Wiley, New York, 1964).

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[6]For the 1930s, see T. Palander, Beiträge zur Standortstheorie (Almqvist och Wicksell, Uppsala, 1935) and E. M. Hoover, Jr., Location Theory and the Shoe and Leather Industries (Harvard University Press, Cambridge, 1937). For the present there are several choices, e.g., H. W. Richardson, Regional Economics: Location Theory, Urban Structure, and Regional Change (Praeger, New York, 1969). An overview may be found in the articles on "Spatial Economics" by E. M. Hoover and L. N. Moses in ~~the~~ International Encyclopedia of the Social Sciences (Macmillan, New York, 1968).

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[7]Of course, a number of disciplines have evolved to deal with spatial problems: network flow theory, traffic flow theory, the linear programming transportation problem, information diffusion models, etc. But none of these directly touches the traditional problems of location theory. ~~In~~ Chapter 7 ~~I~~ generalizes the transportation problem to measure-theoretic form, whereupon it *does* become directly relevant to several of these problems.

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[8]E. S. Dunn, Jr., The Location of Agricultural Production (University of Florida Press, Gainesville, 1954); W. Isard, Location and Space-Economy (M.I.T. Press, Cambridge, 1956), chapter 7; W. Alonso, Location and Land-Use (Harvard ~~UP~~ University Press, Cambridge, 1964).

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9. [9]See the discussion in Alonso, Location and Land-Use, chapter 1 and pages 101-105. The first semi-rigorous argument for the affirmative appears to be Faden, ~~my~~ Essays in Spatial Economics, pp. ~~pages~~ 170-188, though it is couched in equilibrium rather than optimizing terms. This argument is essentially repeated ~~below, page~~ ~~in Section 8.76~~ below on pp. 000-000.

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10. [10]A very simple model exists on the uniform plans with uniform costs in which the optimal distribution appears to be "mixed"; see A. M. Faden, "Inefficiency of the Regular Hexagon in Industrial Location," Geographical Analysis, 1:321-328, October, 1969; see also ~~pages~~ ~~section 9.60~~ below. pp. 000-000.

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11. [11]The terms "discrete" and "continuous" are used throughout this discussion in an intuitive sense. The technically correct terms are "atomic" and "non-atomic", respectively.

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12. [12]A semantic note: The disciplines called regional science, human ecology, theoretical geography, and ekistics are in my view essentially the same as spatial economics or location theory, in the sense that they share a common core of central ideas, though each retains its particular flavor. There is product differentiation in the marketplace of ideas as well as ~~in~~ that of commodities.

92-15

13. [13]As examples of parts not covered in this book are ~~I may mention~~ the control-theoretic models in E. S. Mills, Studies in the Structure of the Urban Economy (Johns Hopkins Press, Baltimore, 1972), and the Löschian comparative statics, the homogeneous Thünen, and the commuting models in Faden, ~~my own~~ Essays in Spatial Economics. ~~chapter 1.~~

14. [14] Equilibrium models may be found ~~in the following~~ throughout chapters 6 and sections: 5.8, ~~6 passion~~, 8.7, 8.8, 9.2, 9.5, 9.6.

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15. [15] A brief survey of algorithms for location theory may be found in A. J. Scott, Combinatorial Programming, Spatial Analysis and Planning (Methuen, London, 1971).

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[16] ~~As already discussed~~, this framework is adequate for descriptive purposes in all ~~of~~ social science. What about natural science? On the whole it remains adequate; but ~~note that~~ the framework is incompatible with relativity theory, since it assumes "absolute" Space and Time. And it is (probably) incompatible with quantum theory, since it allows for complete state descriptions.

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[17] These virtues arise from the fact that ~~one does not have~~ it is unnecessary to impose artificial restrictions to guarantee that utilities are real-valued. Of course, simplicity is in the eye of the beholder. If English spelling were made phonetic, it would be objectively simpler, but not necessarily to someone who had learned to spell the old way.

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[18] Following ~~A. M.~~ Faden, "Inefficiency of the Regular Hexagon, ~~in Industrial Location,"~~ Geographical Analysis, ~~1:~~321–328, ~~October, 1969.~~